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Competition or cooperation in transboundary fish stocks management: Insight from a dynamical model

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ABSTRACT

An idealized system of a shared fish stock associated with different exclusive economic zones (EEZ) is modelled. Parameters were estimated for the case of the small pelagic fisheries shared between Southern Morocco, Mauritania and the Senegambia. Two models of fishing effort distribution were explored. The first one considers independent national fisheries in each EEZ, with a cost per unit of fishing effort that depends on local fishery policy. The second one considers the case of a fully cooperative fishery performed by an international fleet freely moving across the borders. Both models are based on a set of six ordinary differential equations describing the time evolution of the fish biomass and the fishing effort. We take advantage of the two time scales to obtain a reduced model governing the total fish biomass of the system and fishing efforts in each zone. At the fast equilibrium, the fish distribution follows the ideal free distribution according to the carrying capacity in each area. Different equilibria can be reached according to management choices. When fishing fleets are independent and national fishery policies are not harmonized, in the general case, competition leads after a few decades to a scenario where only one fishery remains sustainable. In the case of sub-regional agreement acting on the adjustment of cost per unit of fishing effort in each EEZ, we found that a large number of equilibria exists. In this last case the initial distribution of fishing effort strongly impact the optimal equilibrium that can be reached. Lastly, the country with the highest carrying capacity density may get less landings when collaborating with other countries than if it minimises its fishing costs. The second fully cooperative model shows that a single international fishing fleet moving freely in the fishing areas leads to a sustainable equilibrium. Such findings should foster regional fisheries organizations to get potential new ways for neighbouring fish stock management.

1. Introduction

Exploited fish populations performing transboundary migrations is a common situation around the world, as obviously no fish is stopped at or by any country border. Documented examples include shared small pelagic fisheries in the California current system between Canada, USA and Mexico (Javor et al., 2011; Lo et al., 2011) as well as in the Canary Current System between Morocco, Mauritania, Senegal, Gambia, and Guinea Bissao (Boely et al., 1982; Brochier et al., 2018), which are both East Boarder Upwelling. Reaching fishery agreements between the countries exploiting same fish stock is a prerequisite for a good, equitable management (e.g. Campbell and Hanich, 2015; Pitcher et al., 2002) in waters under national jurisdiction e.g. to avoid fish stock overexploitation, to understand change in stock spatial distribution in

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the context of climate change which may quickly disturb the existing equilibria (Miller and Munro, 2004).

From a theoretical point of view, finding a common supranational policy allowing a sustainable fishing activity simultaneously in all the countries exploiting the same fish stock is not a trivial problem and a topic of high political interest. This issue is usually studied in the literature using the game theory for the economic analysis of international fisheries agreements (e.g. Aguero and Gonzalez, 1996; Ishimura et al., 2012; Pintassilgo et al., 2014). These approaches demonstrated that the full cooperative management by the different countries is necessary to achieve sustainable fisheries, and that the works to optimize fisheries management in the context of climate change (as other issues) need to be conducted in common or at least simultaneously by the concerned countries to be successful (e.g. Ishimura et al., 2012). A key message that emerges from this literature strand is that the self organisation generally leads to over-exploitation of internationally shared fish stocks (Campbell and Hanich, 2015; Pintassilgo et al., 2014; Pitcher et al., 2002). Thus, an international legal framework and regulations must be developed to help the complex system of shared fisheries to avoid the over-exploitation trap (Feeny et al., 1996) and to converge toward the optimum situation of collective maximum sustainable yield.

In this work we use a set of ordinary differential equations (ODE) to explore the situations that can emerge either in the case of independent national fishing fleets (each one fishing only in its own EEZ) in or in the case of an unique international fishing fleet that can freely move between EEZs. In the first case, we consider a management acting on the cost per unit of fishing effort (CUFE) instead of catch quota in the classical approach. The behaviour of decision makers is not explicitly modelled, but is represented by the CUFE applied in each EEZ. Competition occurs when each country tends to minimise CUFE, but we show that fishing agreements can tend to harmonise local CUFEs in order to limit the competition. In the second case, that we called fully cooperative, the CUFE is set constant among the EEZ, and the fishing agreement was assumed to be a negotiation on the share of the international fishing fleet benefits.

Numerous bio-economic models based on ODE were used to explore the optimal harvesting rates in a given fisheries system (e.g. Crutchfield, 1979; Dubey et al., 2002; Merino et al., 2007), but less effort was developed to apply such models to the case of fisheries managed by different governments exploiting the same fish population. Here we assume an ideal situation considering that all the fisheries are effectively regulated (no illegal, unreported, and unregulated (IUU) Fishing (Agnew et al., 2009)). We applied our approach to the case of the North–West Africa fisheries in the South part of the Canary Upwelling System (CUS), using the output of a realistic physical biogeochemical model recently developed for this region (Auger et al., 2016) to describe the environment which forces the fish migrations.

2. Preliminaries

We explore the behaviour of an idealized system of shared fisheries that suits the current knowledge of the ecosystem in the CUS. We consider the fish stock that perform migrations between 12.3°N (Senegal) to 26°N (Morocco). We consider 3 political ensembles within which we assume the fishery regulation could be uniform, the Southern Morocco/Western Sahara (20.77–26°N, hereafter zone 1), Mauritania (16.06–20.77°N, hereafter zone 2) and the Senegambia (Senegal + the Gambia, 12.3–16.06°N, hereafter zone 3) (Fig. 1).

The small pelagic fish carrying capacity from Senegal to South Morocco was estimated to ∼10 million tons, including the main fished species namely *Sardina pilchardus, Sardinella aurita and S. maderensis, Trachurus spp., Scomber japonicus, Caranx rhonchus*

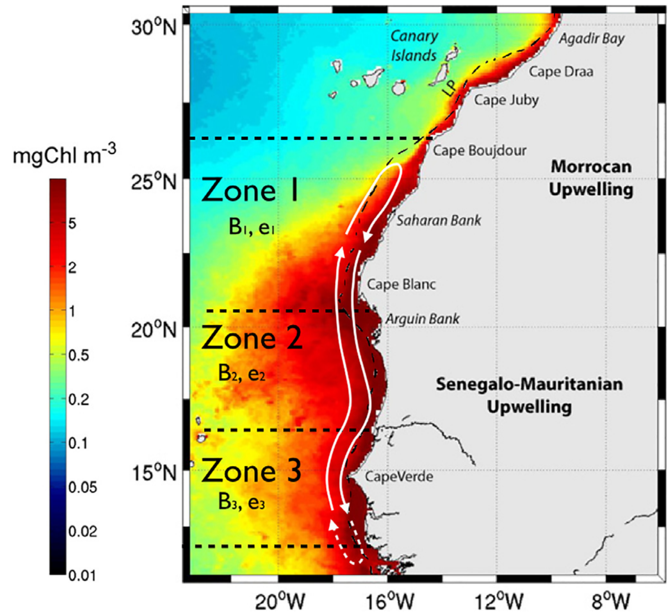

Fig. 1. Illustration of the shared small pelagic fisheries in the South part of the West African Eastern Boundary Upwelling System. The colourbar indicates the mean chlorophyll 'a' (1998–2009) which was used as a proxy for the small pelagic fish carrying capacity. The dotted black line shows the limit of the continental shelf which shelter small pelagic fish in this area. The white arrows indicate the seasonal migrations performed by the round sardinella, the main exploited small pelagic fish species in the region (Boely et al., 1982). The dotted white arrow corresponds to the less documented part of the migration (to Guinea Bissau). B_i and e_i are the biomass and fishing effort in each zone, respectively, i.e., (1) the Southern Morocco/Western Sahara, (2) Mauritania, (3) Senegal and Gambia. Adapted from Auger et al. (2016). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

(FAO, 2012). The carrying capacity is the maximum biomass of fish that can feed and grow in this region in average climatic conditions (1998–2009), according to available food, here estimated by phyto-plankton for the small pelagic fish considered (Aguero and Gonzalez, 1996).

Each of these species has distinct environmental preferendum and thus migration behaviour (Boely et al., 1982), but all together these migrations can be represented by as an ideal free distribution of the total small pelagic fish biomass according to the environment carrying capacity. The fish annual growth (r) was set to 0.88 year^{-1}, as an average of small pelagic fish growth rates in West Africa (FAO, 2012).

According to observations in the CUS, the small pelagic fish habitat is restricted to the continental shelf (Brehmer, 2004; Brehmer et al., 2006). We computed the surface of the continental shelf, delimited offshore by the 200 m isobath, from the bathymetry used by Auger et al. (2016). We found that the part of the Southern Morocco/Western Sahara considered (zone 1) has a shelf of 41 472 km^2, the Mauritania shelf (zone 2) is 22 720 km^2 and the Senegambia shelf (zone 3) is 15 040 km^2. The physical - biogeochemical ocean simulation in this region (Auger et al., 2016) provides the mean annual plankton biomass: 255 014 tons in zone 1, 105 575 tons in zone 2 and 51 632 tons in zone 3. To estimate the carrying capacity in each defined zone, we distributed the total carrying capacity estimated (10 million tons; FAO, 2012) according to the average plankton biomass density (food of the small pelagic fish) in the small pelagic fish habitat of each zone. We found the average local carrying capacity $k_1 = 104$ tons/km^2 in south Morocco (zone 1), $k_2 = 45$ tons/km^2 in Mauritania (zone 2) and $k_3 = 160$ tons/km^2 in Senegambia (zone 3). The total carrying capacity found was greater in zone 1 ($K_1 = 4.3$ million tons), than in zone 2 ($K_2 = 3.3$ million tons) and lowest in zone 3 ($K_3 = 2.4$ million tons).

The dynamic of the system is modelled by a system of 6 ordinary differential equations (1). We assume that fish are performing rapid migrations across zone 1, 2 and 3, such that their average annual biomass $B_1(t)$, $B_2(t)$ and $B_3(t)$ in each zone follow the ideal free distribution according to the carrying capacity K_1, K_2 and K_3. Fish movement is considered as a rapid phenomenon (Brehmer et al., 2000) compared to the growth rate of their population and compared to the fishing mortality rate. As a result, at the fast equilibrium there is a fixed proportion of the total fish abundance in each zone. Indeed, it is generally accepted that small pelagic fish migration is due to environment, and not linked to fish density nor local policy in fishing strategy. The rate of migration is quantified by the parameter $''a''$ corresponding to the change in biomass density due to fish movement between the zones at the fast time scale ($[a]$ = million tons/10 days). Two contrasted cases were considered for the movement of the fishing boats. In one case, we assume that the fishing boats cannot freely cross the national marine borders to lead fishing activities, and in each zone, fishing vessels/canoes are committed to a specific policy that affects the Cost per Unit of Fishing Effort (CUFE). Thus, in this case we also assume that the fishing investments in each country are independent. In the second case, we assume that fishing boats can freely cross the national marine borders and that the CUFE policy is uniform in the 3 zones.

The fishing efforts in each country are denoted $e_1(t)$, $e_2(t)$ and $e_3(t)$. The fishing effort is expressed in surface prospected by the fishing boats (10^4 km^2). If the surface fished exceeds the surface of the area, it means that the fishing boats operate more than one time in each km^2. As an indicative range, in this idealized model we consider the fishing effort in a zone i of surface S_i should not exceed $365 \times S_i$, corresponding to each km^2 of zone i being fished daily. Let us assume the fishing effort in each zone is controlled through a local policy impacting the cost per unit of effort. Furthermore, it is considered that each country has a very high level of governance and military means which ensures the absence of illegal fishing activities ignoring the maritime borders. There is no precise data on real values of fishing efforts in the three countries, indeed even the Copace/FAO working group from the united nations is still looking for estimations of the fishing efforts and to find an approach for their standardization. Thus in this work the only alternative approach was to propose an arbitrary set of values, based on local fisheries experiences.

By analysing the average fuel consumption, the “absolute” CUFE can be estimated. Indeed, on the basis of the fuel consumption of medium size fishing vessels ($\sim$30 m), Sala et al. (2011) found a mean fuel consumption of $\sim$130 l/h during a fishing operation, which occur at $\sim$4–5 km/h. For this kind of boat, we assume that the mean radius of fish detection (by instruments and/or direct visual) is 50 m around the boat. Based on these assumptions, we estimate an average fuel consumption of $\sim$300 l/km^2 fished. Considering the fuel price (f) at 1 US$/l, then our estimate of the minimum cost per unit of effort for these boats is 300 US$. This is an underestimate of the total cost because we neglect the investment for the boat maintenance and the crew. However we will use this value as a basis in order to estimate whether the governance in each zone must provide subsidies or tax the local fisheries in order to reach the maximum sustainable yield at the regional scale. Typical subsidies that impact the CUFE are reduced price of fuel for fishing activity. In this paper, we reflect the impact of tax or subsidies by increasing or reducing the CUFE. The rate of re-investing benefice into fisheries (ϕ) was arbitrary set to 1, which means that all the benefits are invested into fishing effort (in the same zone). Symmetrically, the losses are directly translated into reducing the fishing effort in the same zone.

The fish catchability coefficient (q) is defined here as the fraction of the average fish density that is harvested per each unit effort. For example, if the fish density is 1 ton/km^2, and the catchability 0.5, then the harvest is 0.5 tons. Catchability is the parameter that is the harder to estimate from real expert knowledge in the context of strong model assumptions on prospection, fishing techniques and fish density distribution (uniform within a zone). Thus, we fix the order of magnitude of this parameter arbitrarily to 0.01 such that the order of magnitude of the fishing efforts remained in a realistic range (see Table 1 and 2).

The fish ex-vessel price is very highly variable on the coast, according the seasonal fluctuations in abundances (Failler, 2014). However, the distant water fishing vessels are more concerned by the global ex-vessel price which is more constant, around 500 US$/tons (Swartz et al., 2012).

We shall now present two models corresponding to different strategy that may be applied by decision makers for the exploitation of the shared fish stock. The first model is referred to as competitive model, with a main assumption that each national fleet only operates in its national fishing zone. The second model is referred as the fully cooperative model, with the main assumption that the three countries collaborate and have an agreement to constitute a common fleet operating in the three fishing zones.

2.1. Competitive model

We now present the equations of the complete competitive model:

$$\begin{cases} \dfrac{dB_1}{d\tau} = \left(\dfrac{aB_2}{K_2} - \dfrac{aB_1}{K_1}\right) + \varepsilon\left(-\dfrac{qB_1e_1}{S_1} + rB_1\left(1 - \dfrac{B_1}{K_1}\right)\right) \\ \dfrac{dB_2}{d\tau} = \left(\dfrac{aB_1}{K_1} + \dfrac{aB_3}{K_3} - 2\dfrac{aB_2}{K_2}\right) + \varepsilon\left(-\dfrac{qB_2e_2}{S_2} + rB_2\left(1 - \dfrac{B_2}{K_2}\right)\right) \\ \dfrac{dB_3}{d\tau} = \left(\dfrac{aB_2}{K_2} - \dfrac{aB_3}{K_3}\right) + \varepsilon\left(-\dfrac{qB_3e_3}{S_3} + rB_3\left(1 - \dfrac{B_3}{K_3}\right)\right) \\ \dfrac{de_1}{d\tau} = \varepsilon\left(\dfrac{\phi}{c_1}\left(\dfrac{pqB_1e_1}{S_1} - c_1e_1\right)\right) \\ \dfrac{de_2}{d\tau} = \varepsilon\left(\dfrac{\phi}{c_2}\left(\dfrac{pqB_2e_2}{S_2} - c_2e_2\right)\right) \\ \dfrac{de_3}{d\tau} = \varepsilon\left(\dfrac{\phi}{c_3}\left(\dfrac{pqB_3e_3}{S_3} - c_3e_3\right)\right). \end{cases} \quad (1)$$

In order to reduce the complete model, we apply aggregation method (Auger et al., 2008). We set $\varepsilon = 0$ in system (1) and the simple calculation leads to the following fast equilibrium

$$B_1^* = \alpha_1 B, \quad B_2^* = \alpha_2 B, \quad B_3^* = \alpha_3 B,$$

in which

$$\alpha_1 = \frac{K_1}{K}, \quad \alpha_2 = \frac{K_2}{K}, \quad \alpha_3 = \frac{K_3}{K},$$

$$K = K_1 + K_2 + K_3,$$

α_1, α_3, α_3 represent the proportions of fish biomass in zone 1, 2, 3 at the fast equilibrium respectively, and $B(t) = B_1(t) + B_2(t) + B_3(t)$ is the total fish biomass at the slow time scale $t = \varepsilon\tau$. Substituting the fast equilibrium into the equations of complete model,

Table 1
List of model parameters identified for small pelagic fish fisheries in North–West Africa (∼ 12.3–26°N). Units were chosen such that most parameters are of the order of one (following the requirement for the aggregation method to be applied), except fish price and catchability coefficient.

Symbol	Meaning	Unit	Value
Time-scale parameters			
ε	Ratio between fast (∼ 10 days) and slow (∼ 1 year) processes (t/τ). Must be $\sim < 0.1$ so that the aggregation method can be used (Auger et al., 2008)	No dimension	10/365
Environmental and biological parameters for the exploited small pelagic fish			
S_1	Surface of fish habitat in zone 1: continental shelf from 20.77°N to 26°N (South Morocco)	10^4 km^2	4.1472 (a)
S_2	Surface of fish habitat in zone 2: continental shelf from 16.06°N to 20.77°N (Mauritania)	10^4 km^2	2.2720 (a)
S_3	Surface of fish habitat in zone 3: continental shelf from 12.3°N to 16.06°N (Senegambia)	10^4 km^2	1.5040 (a)
K_1	Total small pelagic fish carrying capacity in zone 1	Megaton (10^6 tons)	4.3 (b)
K_2	Total small pelagic fish carrying capacity in zone 2	Megaton (10^6 tons)	3.3 (b)
K_3	Total small pelagic fish carrying capacity in zone 3	Megaton (10^6 tons)	2.4 (b)
a	Fish migration rate. Define the "speed" at which the fish tends to the ideal free distribution according to the local carrying capacities	Megatons/10 days	1 (c)
r	Fish growth rate (average among small pelagic species in West Africa; FAO (2012))	year^{-1}	0.88
Economic/efficiency parameters for the small pelagic fish fisheries			
f	Price of the fuel	Millions \$/1000 m^3 (=\$/l)	1
q	Fish catchability. Represent the fraction of the average fish biomass in given area catch by a fishing vessel trawling on the entire area	No dimension	0.01 (c)
p	Fish price. World average ex-vessel price for small pelagic fish	Millions \$/Megatons	500 (d)
c_i	Cost for fishing effort in zone i. The value given here correspond to the minimum to ensure boat maintenance, fuel consumption and crew. National policy may increase or reduce these costs (subsidies or tax)	Millions \$/[fishing effort] (or 10^6\$ 10^4 km^{-2})	3 × f (e)
ϕ	Rate of re-investing benefits in fisheries	1/year	1 (c)
β	1st Parameter for the migration of boats in cooperative model: ideal free distribution when B is large	τ / [B/S]	1
β_0	2nd Parameter for the migration of boats cooperative in model: uniform distribution when B is small	τ / [S]	1

(a) Surface of the continental shelf, from coast to 200 m isobat (calculated from Etopo2). (b) Total small pelagic Carrying capacity estimated to ∼ 10 million tons (FAO (2012), distributed between the zone 1 and 3 according to the mean local plankton density estimated (Eric Machu, IRD, pers. Com). (c) Arbitrary. (d) Swartz et al. (2012). (e) Estimated from the observations of Sala et al. (2011).

Table 2
List of states variables used in the complete model.

Symbol	Meaning	Unit	Range
B_i	Fish biomass in zone i	Megatons	0–K_i[a]
e_i	Fishing effort in zone i, expressed in unit area fished	10^4 km^2	0–65 × S_i[b]

[a] K_i = fish carrying capacity in zone i, see Table 1.
[b] There is no maximum value for the fishing effort in the model; however we consider that the fishing effort in zone i should be reasonably under 365 × S_i, corresponding to each km^2 in the area being fished daily.

the aggregated competitive model reads as follows

$$\begin{cases} \dfrac{dB}{dt} = \left(-\dfrac{q\alpha_1}{S_1}e_1 - \dfrac{q\alpha_2}{S_2}e_2 - \dfrac{q\alpha_3}{S_3}e_3\right)B + rB\left(1-\dfrac{B}{K}\right) \\ \dfrac{de_1}{dt} = \dfrac{\phi}{c_1}\left(\dfrac{pq\alpha_1}{S_1}e_1B - c_1e_1\right) \\ \dfrac{de_2}{dt} = \dfrac{\phi}{c_2}\left(\dfrac{pq\alpha_2}{S_2}e_2B - c_2e_2\right) \\ \dfrac{de_3}{dt} = \dfrac{\phi}{c_3}\left(\dfrac{pq\alpha_3}{S_3}e_3B - c_3e_3\right). \end{cases} \quad (2)$$

To analyse the dynamical behaviour of aggregated competitive model, we will survey stability of non-negative equilibrium points $(B^*, e_1^*, e_2^*, e_3^*)$ of the Eq. (2). We see right that two points $O(0, 0, 0, 0)$ and $A(K, 0, 0, 0)$ are non-negative equilibrium points of the equation. By linearized principle then the point O is unstable. The stability of point A is stated in the following proposition.

Proposition 2.1. *If* $\overline{B}_i = \dfrac{c_iS_i}{pq\alpha_i} \geq K$ $(\Leftrightarrow \dfrac{c_iS_i}{pqK_i} \geq 1)$ *for all* $i = \overline{1,3}$ *then the point A is globally asymptotically stable and* $\lim_{t\to\infty} e_i(t) = 0,\ i = \overline{1,3}$.

Proof. See Appendix A. □

When the assumption of Proposition 2.1 is infringed, i.e., there exists $\overline{B}_i < K$. By linearized principle, point A is unstable. However, the system (2) has additional non-negative equilibrium points. Without loss of generality, we assume $\overline{B}_i < K$ for all $i = \overline{1,3}$. With this assumption we have covered all non-negative equilibrium points of system (2). Concretely, we have the following cases

- Case 1: If $\overline{B}_i \neq \overline{B}_j$ for $i \neq j$ then the system has to add equilibrium points
$$P_1\left(\overline{B}_1, \frac{rS_1(K-\overline{B}_1)}{Kq\alpha_1}, 0, 0\right),\ P_2\left(\overline{B}_2, 0, \frac{rS_2(K-\overline{B}_2)}{Kq\alpha_2}, 0\right),$$
$$P_3\left(\overline{B}_3, 0, 0, \frac{rS_3(K-\overline{B}_3)}{Kq\alpha_3}\right).$$
- Case 2: If only there exists a pair $\overline{B}_i = \overline{B}_j \neq \overline{B}_k$ for i, j, k different then the system has to add equilibrium point P_k (in case 1) and a set of equilibrium points $\Gamma_i = \{M_i\}$ where $B = \overline{B}_i, e_k = 0$ and $e_i = e_i^*, e_j = e_j^*$ satisfy $\frac{q\alpha_i}{S_i}e_i^* + \frac{q\alpha_j}{S_j}e_j^* = r(1 - \frac{\overline{B}_i}{K})$.
- Case 3: If $\overline{B}_1 = \overline{B}_2 = \overline{B}_3 =: \overline{B}^*$ then the system has to add a set of equilibrium points, denoted by $\Gamma = \{M(\overline{B}^*, e_1^*, e_2^*, e_3^*)\}$, in which e_1^*, e_2^*, e_3^* satisfy $\frac{q\alpha_1}{S_1}e_1^* + \frac{q\alpha_2}{S_2}e_2^* + \frac{q\alpha_3}{S_3}e_3^* = r(1 - \frac{\overline{B}^*}{K})$.

To investigate stability of the non-negative equilibrium points of system (2) in each the above case, we denote $\mathbb{R}_+^4 = \{x \in \mathbb{R}^4 : x_m \geq 0 \text{ for } m = \overline{1,4}\}$, $\mathbb{R}_{++}^4 = \{x \in \mathbb{R}^4 : x_m > 0 \text{ for } m = \overline{1,4}\}$ and $\mathbb{R}_{+m}^4 = \{x \in \mathbb{R}_+^4 : x_m > 0\}$ with $m = \overline{1,4}$. Through analyse stability of the equilibrium point, we obtain asymptotic behaviour for solution of system (2) in the following theorem.

Theorem 2.2. *Assume that* $\overline{B}_i = \dfrac{c_iS_i}{pq\alpha_i} < K$ $(\Leftrightarrow \dfrac{c_iS_i}{pqK_i} < 1)$ *for all* $i = \overline{1,3}$*. Then the following assertions hold.*

a) *If* $\overline{B}_i \neq \overline{B}_j$ *for* $i \neq j$ *and* $\overline{B}_i$ *is the smallest then point* P_i *is globally asymptotically stable, other points are unstable and* $\lim_{t\to\infty} e_k(t) = 0,\ k \neq i.$
b) *If there exists a pair* $\overline{B}_i = \overline{B}_j \neq \overline{B}_k$ *for i, j, k different then there exists at least one fishing effort variable tend to zero. Moreover, if* $\overline{B}_k$ *is the smallest then* P_k *is globally asymptotically stable and the equilibrium points in* Γ_i *are unstable. Otherwise,* P_k *is unstable and* Γ_i *is attraction set for all solution trajectories of the system (2).*
c) *If* $\overline{B}_1 = \overline{B}_2 = \overline{B}_3 =: \overline{B}^*$ *then* Γ *is attraction set for all solution trajectories of the system (2). Moreover, the solutions of the system with initial values in* $\mathbb{R}^4_{++}$ *will converge to equilibrium points in* $\Gamma \cap \mathbb{R}^4_{++}$.

Proof. See Appendix A. □

If there exists i such that $\overline{B}_i \geq K$ then Case 3 doesn't exist, i.e., the system (2) has not any equilibrium point in the set Γ. By the same argument of Theorem 2.2 we obtain the following corollary.

Corollary 2.3. *Assume that there exists* $i, j \in \{1, 2, 3\}$ *such that* $K \in (\overline{B}_i, \overline{B}_j]$, *then solution of the system (2) always contains at least a component function being* $e_j(t)$ *satisfy* $\lim_{t\to\infty} e_j(t) = 0$.

Proposition 2.1, Theorem 2.2, Corollary 2.3 has given us asymptotic behaviour of the solution of system (2) in all the cases. Summary, we have the following assertions.

Proposition 2.4. *Putting* $\overline{B}_i = \dfrac{c_i S_i}{pq\alpha_i}$ *with* $i = \overline{1,3}$. *Then,*

a) *If* $\overline{B}_i \geq K$ $\left(\Leftrightarrow \dfrac{c_i S_i}{pqK_i} \geq 1\right)$ *with some* $i \in \{1, 2, 3\}$ *then* $\lim_{t\to\infty} e_i(t) = 0$.
b) *If* $\max\{\overline{B}_1, \overline{B}_2, \overline{B}_3\} < K$ *and* $\overline{B}_i > \min\{\overline{B}_1, \overline{B}_2, \overline{B}_3\}$ *with some i then* $\lim_{t\to\infty} e_i(t) = 0$.

Proposition 2.4 shows the conditions for the extinction and permanence of fishing effort. Hence, fishery model will be permanent if the parameters of model have to satisfy the conditions in Case 3, i.e. $\overline{B}_1 = \overline{B}_2 = \overline{B}_3 = \overline{B}^*$. When the solutions of model tend to equilibria in Γ, we will find out the conditions for the maximum of total catch of the fishery at the equilibrium. The catch per unit of time at equilibrium of the slow aggregated model reads as follows

$$H := \sum_{i=1}^{3} q_i \overline{B}^* e_i^* = \left(\frac{q\alpha_1}{S_1}e_1^* + \frac{q\alpha_2}{S_2}e_2^* + \frac{q\alpha_3}{S_3}e_3^*\right)\overline{B}^*$$

where $q_i = \frac{q\alpha_i}{S_i}$ be a local catchability in zone i. Since $(\overline{B}^*, e_1^*, e_2^*, e_3^*) \in \Gamma$ then $H = (\frac{q\alpha_1}{S_1}e_1^* + \frac{q\alpha_2}{S_2}e_2^* + \frac{q\alpha_3}{S_3}e_3^*)\overline{B}^* = r(1 - \frac{\overline{B}^*}{K})\overline{B}^*$. Thus H gets maximal value $H^* = r\frac{K}{4}$ when

$$\frac{K}{2} = \overline{B}^* = \frac{c_1S_1}{pq\alpha_1} = \frac{c_2S_2}{pq\alpha_2} = \frac{c_3S_3}{pq\alpha_3} \Leftrightarrow \frac{c_1S_1}{K_1} = \frac{c_2S_2}{K_2} = \frac{c_3S_3}{K_3} = \frac{pq}{2}. \tag{3}$$

The optimal attraction set is $\Gamma^* = \{M(\overline{B}^* = \frac{K}{2}, e_1^*, e_2^*, e_3^*)\}$, where e_1^*, e_2^*, e_3^* satisfy

$$\frac{q\alpha_1}{S_1}e_1^* + \frac{q\alpha_2}{S_2}e_2^* + \frac{q\alpha_3}{S_3}e_3^* = \frac{r}{2}. \tag{4}$$

Figs. 2 and 3 present numerical simulations of the complete model with two cases, only one fishery is surviving and coexistence of the three fisheries. On Fig. 2, we superpose the three projections of the trajectory of the complete model in the planes (B_i, e_i) with $i = \overline{1,3}$.

2.2. Fully cooperative model

In cooperative model, we assume the nations have a common agreement to constitute a common fishing fleet, each country contributing to the global fishing effort. The boats are allowed to move and to catch fish in the three fishing zones. Furthermore, it is assumed that the fishing vessels move between the different fishing zones with respect to local fish biomass. In this way, boats migration rates are supposed to be fish stock dependent. The larger is the fish biomass in a fishing zone, the less boats leave this area per unit of time. In other words, it is assumed that fleets remain longer times in zones where fish is more abundant. We also suppose that all costs are equal, i.e. $c_1 = c_2 = c_3 = c$. Then the equations of complete cooperative model read as follows:

$$\begin{cases} \dfrac{dB_1}{d\tau} = \left(\dfrac{aB_2}{K_2} - \dfrac{aB_1}{K_1}\right) + \varepsilon\left(-\dfrac{qB_1e_1}{S_1} + rB_1\left(1 - \dfrac{B_1}{K_1}\right)\right) \\ \dfrac{dB_2}{d\tau} = \left(\dfrac{aB_1}{K_1} + \dfrac{aB_3}{K_3} - 2\dfrac{aB_2}{K_2}\right) + \varepsilon\left(-\dfrac{qB_2e_2}{S_2} + rB_2\left(1 - \dfrac{B_2}{K_2}\right)\right) \\ \dfrac{dB_3}{d\tau} = \left(\dfrac{aB_2}{K_2} - \dfrac{aB_3}{K_3}\right) + \varepsilon\left(-\dfrac{qB_3e_3}{S_3} + rB_3\left(1 - \dfrac{B_3}{K_3}\right)\right) \\ \dfrac{de_1}{d\tau} = \left(\dfrac{e_2}{\beta\frac{B_2}{S_2} + \beta_0S_2} - \dfrac{e_1}{\beta\frac{B_1}{S_1} + \beta_0S_1}\right) + \varepsilon\left(\dfrac{\phi}{c}\left(\dfrac{pqB_1e_1}{S_1} - ce_1\right)\right) \\ \dfrac{de_2}{d\tau} = \left(\dfrac{e_1}{\beta\frac{B_1}{S_1} + \beta_0S_1} + \dfrac{e_3}{\beta\frac{B_3}{S_3} + \beta_0S_3} - 2\dfrac{e_2}{\beta\frac{B_2}{S_2} + \beta_0S_2}\right) \\ \qquad + \varepsilon\left(\dfrac{\phi}{c}\left(\dfrac{pqB_2e_2}{S_2} - ce_2\right)\right) \\ \dfrac{de_3}{d\tau} = \left(\dfrac{e_2}{\beta\frac{B_2}{S_2} + \beta_0S_2} - \dfrac{e_3}{\beta\frac{B_3}{S_3} + \beta_0S_3}\right) + \varepsilon\left(\dfrac{\phi}{c}\left(\dfrac{pqB_3e_3}{S_3} - ce_3\right)\right). \end{cases} \tag{5}$$

Using the similar argument, we obtain the same fast equilibrium $B_1^* = \alpha_1B, B_2^* = \alpha_2B, B_3^* = \alpha_3B$ and

$$e_i^* = \frac{\beta\frac{\alpha_i}{S_i}B + \beta_0S_i}{\beta\mu B + \beta_0\nu}e \quad i = \overline{1,3},$$

where $\mu = \sum_{i=1}^3 \frac{\alpha_i}{S_i}$, $\nu = \sum_{i=1}^3 S_i$ and $e(t) = e_1(t) + e_2(t) + e_3(t)$. So we have aggregated cooperative model

$$\begin{cases} \dfrac{dB}{dt} = -\dfrac{q(\beta\gamma B + \beta_0\delta)}{\delta(\beta\mu B + \beta_0\nu)}Be + rB\left(1 - \dfrac{B}{K}\right) \\ \dfrac{de}{dt} = \dfrac{\Phi}{c}\left(\dfrac{pq(\beta\gamma B + \beta_0\delta)}{\delta(\beta\mu B + \beta_0\nu)}Be - ce\right). \end{cases} \tag{6}$$

in which

$$\gamma = \frac{(\alpha_1S_2S_3)^2 + (\alpha_2S_1S_3)^2 + (\alpha_3S_1S_2)^2}{\delta},$$

$$\delta = S_1S_2S_3.$$

The dynamical behaviour of (6) is similar to system (9) in Moussaoui et al. (2011). The system (6) has three equilibria (0, 0), (K, 0) and (B^*, e^*), where

$$B^* = \frac{-pq\beta_0\delta + c\delta\beta\mu + \sqrt{(pq\beta_0\delta - c\delta\beta\mu)^2 + 4pq\beta\gamma c\delta\beta_0\nu}}{2pq\beta\gamma} > 0,$$

$$e^* = r\left(1 - \frac{B^*}{K}\right)\frac{\delta(\beta\mu B^* + \beta_0\nu)}{q(\beta\gamma B^* + \beta_0\delta)}.$$

Equilibrium (0, 0) is always a saddle node. We put $\sigma = r\delta(\beta\mu B^* + \beta_0\nu)^2 + Kq\beta\beta_0e^*(\gamma\nu - \delta\mu)$. According to values of parameters, we have three cases:

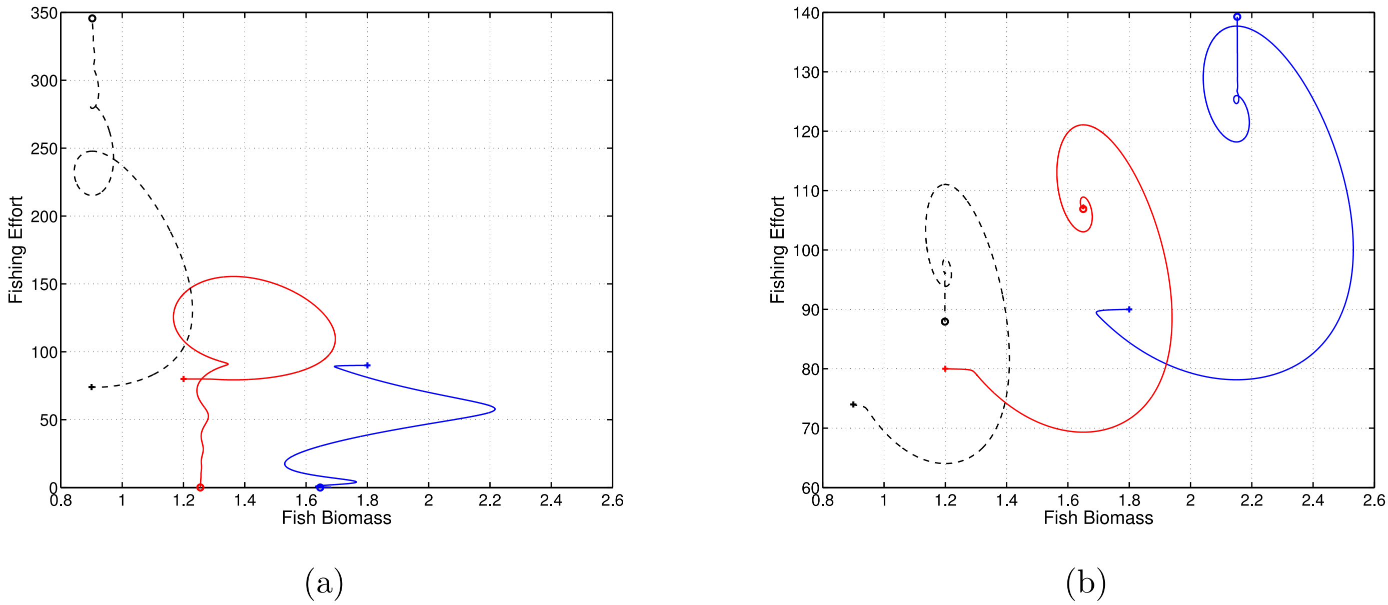

Fig. 2. Numerical simulations of the complete competitive model, phase portraits with initial values $B_1 = 1.8,\ B_2 = 1.2,\ B_3 = 0.9,\ e_1 = 90,\ e_2 = 80,\ e_3 = 74$ and parameters values are in Table 1. Blue, red and dotted lines correspond with the orbit of zone 1, 2 and 3, respectively. (a): Fishery 3 survives, fisheries 1 and 2 go extinct when the cost per unit of effort among the three zones are uniform, $c_1 = c_2 = c_3 = 300$. (b): Coexistence of the 3 Fisheries is stable when total catch gets maximum value, $c_1 = 259.21$, $c_2 = 363.12,\ c_3 = 398.94$. The biomass are in million tons and the fishing effort in 10^4 km^2 prospected each year; the costs of fishing effort are expressed in US\$/km^2 prospected. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

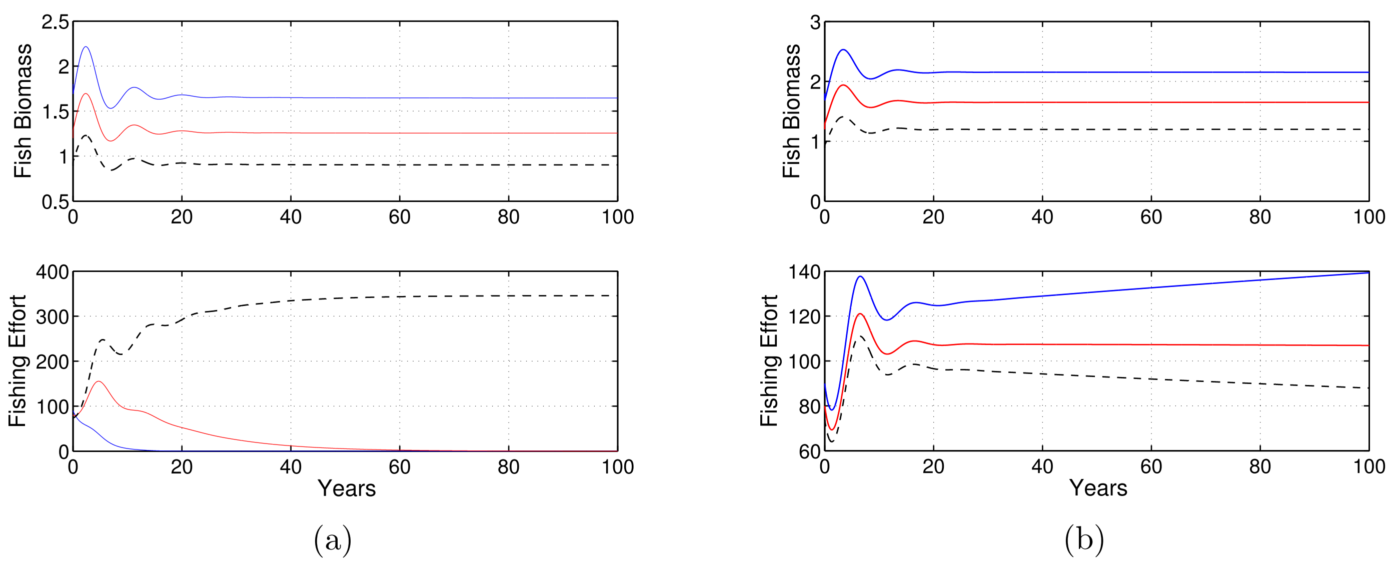

Fig. 3. Numerical simulations of the complete competitive model, time series. The initial values and parameters values are used in Fig. 2. Blue, red and dotted lines correspond with the trajectory of zone 1, 2 and 3, respectively. (a): Fishery 3 survives, fisheries 1 and 2 go extinct. (b): Coexistence of the 3 Fisheries. The biomass are in million tons and the fishing effort in 10^4 km^2 prospected each year. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

- If $B^* > K$ then $(B^*, e^*) < 0$ and $(K, 0)$ is a stable node.
- If $B^* < K$ and $\sigma > 0$ then $(K, 0)$ is a saddle node and (B^*, e^*) is globally asymptotically stable.
- If $B^* < K$ and $\sigma < 0$ then $(K, 0)$ is a saddle node, (B^*, e^*) is unstable and system (6) has a limit cycle.

The catch per unit of time at equilibrium reads

$$\overline{H}^* = \frac{q(\beta\gamma B^* + \beta_0\delta)}{\delta(\beta\mu B^* + \beta_0\nu)} B^* e^* = rB^*\left(1 - \frac{B^*}{K}\right)$$

Thus $\overline{H}^*$ has maximum equal to $r\frac{K}{4}$ when $B^* = \frac{K}{2}$, i.e. $c = c^* = \dfrac{Kpq(K\beta\gamma + 2\beta_0\delta)}{2\delta(2\beta_0\nu + K\beta\mu)}$.

3. Discussion

The mathematical model provides some interesting insights about the dynamic of international shared fish stock. First, we interpret the main results of the mathematical model in terms of fisheries management and compare with the results obtained from other studies. Second, we discuss how this methodology may evolve toward an operational model that can be used in the frame of regional fishing agreements by decision makers and managers.

Table 3
Cost per Unit of Fishing Effort (CUFE) for a standardized industrial fishing boat: model prediction for maximum cost for sustainable fishery and optimal costs for maximizing total catch according to parameters set provided in Table 1; the cost estimated are based on fuel consumption, boat maintenance and crew costs are fixed at 300 US\$/km^2 for the three zones, see Table 1. Unit: cost/km^2 prospected/fished in zone 1, 2, and 3 (must be divided by hundred to convert into millions US\$/10^4 km^2, the unit used in Table 1).

Zones	Maximum cost for sustainable fisheries $c_{i_{max}}$ (\$/km^2)	Optimal cost for maximising total catch $c_{i_{opt}}$ (\$/km^2)
1	518.42	259.21
2	726.23	363.12
3	797.87	398.94

3.1. Interpretation of the main mathematical results

3.1.1. Independent national fleet: shared fisheries as a competition with only one winner in the general case

In each country there is a threshold, maximum value for the CUFE above which the fishery is not viable (Proposition 2.4.a). The model predicted that this value is defined by the relationship:

$$c_{i_{max}} = pqk_i$$

where k_i is the local density of carrying capacity:

$$k_i = K_i/S_i$$

Thus, the higher the carrying capacity density in a given country, the higher cost per unit of effort can be afforded by the fishery. However, if the cost exceeds this threshold value then the fishery activity generates not enough revenue and tends to disappear. For the West Africa case study the threshold costs are given in Table 3, according to the parameters set in Table 1. Under the hypothesis of an absence of management policies and or regulation measures, then for a given type of fishing vessel the CUFE among the three zones would be uniform, solely relying on fuel consumption, gears and vessel/canoe maintenance, and crew costs (Table 1). In this case, the ratio c_i/k_i for zones 1, 2, and 3 is 2.9, 2.1 and 1.9, respectively. Thus in the long term the only surviving fishery would occur in zone 3, in which there is the highest carrying capacity density. This latter case is illustrated in Figs. 2a and 3 a. However, given the different nature of the fisheries in the three countries it is very unlikely to have equals CUFE (see subsection "Toward an operational model of shared fisheries" of the section discussion).

Second, the mathematical analysis also showed that in general, if the average fish biomasses are different between the three countries, only one fishery may actually survive the competition, the one located in the country where the ratio between the CUFE and the carrying capacity density (c_i/k_i) is the lowest (Theorem 2.2.a).

The three countries must find an agreement to ensure sustainable fishing in their respective area. There is only one possible agreement that ensures a sustainable fishing activity in the three countries simultaneously, which is to have equal c/k ratio (Theorem 2.2.c). This case is illustrated in Figs. 2b and 3b. Furthermore, in order to maximize the total catch among the three countries, the CUFE should be set according to the local carrying capacity density (Eq. (3)):

$$c_{i_{opt}} = \frac{pqk_i}{2}.$$

When applying the optimal costs, the trajectory tends towards one of the stable equilibria according to the initial conditions. There are numerous stable equilibriums possible with different distribution of the fishing effort among countries (Fig. 4). Although it was not demonstrated, the sensitivity test with random distribution of the initial fishing effort showed that the initial fishing effort in a given area was significantly correlated with the effort at equilibrium ($R = 0.7$, $p < 0.005$). This suggests that applying the optimal CUFE policy at a given time may tend to stabilize the current distribution of fishing effort, while the amplitude can be modulated, and over-all benefits of the fisheries might increase.

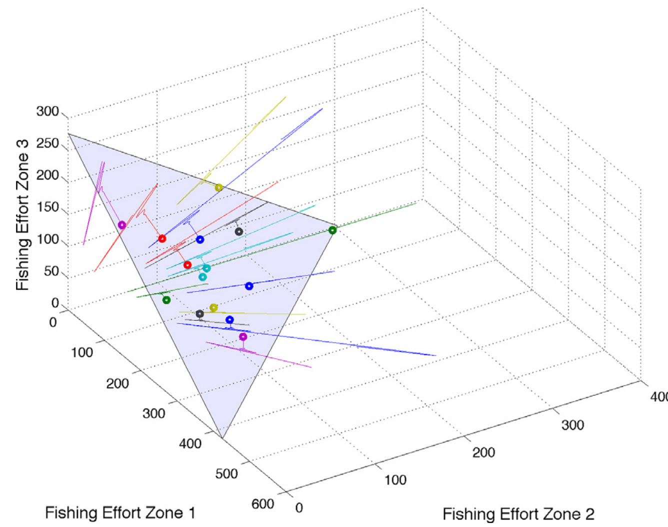

Fig. 4. Tri-dimensional view of the positive part of the fishing efforts in the three countries (e1, e2, and e3) in the case of optimal fishing cost. The blue plane corresponds to the ensemble of optimal equilibria (Eq. (4)). Each coloured level trajectory corresponds to a different set of initial conditions, with the circle indicating the equilibrium reached at $t = 100$ (slow time). Each trajectory converges toward a specific point of the plane, according to the initial conditions. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

However, except in case c) of Theorem 2.2, the competitive model mainly leads to fleet exclusion. As soon as the $\overline{B}_i$ are not exactly equal, we fall in case a) of Theorem 2.2 where a single fleet can survive with extinction of two other fleets in the long term. Indeed, as soon as the three $\overline{B}_i$ are not strictly equal, trajectories are going to move in the direction of one of the exclusion equilibrium which is stable, case a). Maintaining the system in case c) of Theorem 2.2 would assume to change frequently parameters, which seems not so easy in practice. Thus, it must be highlighted that the situation for optimal regional harvesting is a very particular case of the dynamical system (Theorem 2.2, case c). Any small change in the fish biomass distribution (e.g. from climatic fluctuation) or uncontrolled fishing effort (e.g. illegal fishing) may cause the system to switch on a trajectory of the more general case where one fishery takes the advantage on the others (Theorem 2.2 case a). Given the inevitable uncertainties on fish stock evaluations and fishing effort control, it is not realistic for three countries to maintain the exact optimal cost as calculated here. However, the optimal cost could be regularly updated according to the continuous monitoring of the fishing effort and the fish stock states in the three countries (e.g. after one month or one year). Note that the sensibility test showed that it is much more important to have a good estimation of the fishing effort in the three countries than the estimate of the fish biomass. The regular updating of the local optimal fishing costs could allow the three countries to collectively adjust the policies (e.g. tax) to get the total catch closest to the maximum sustainable yield.

If the case of international agreement to tune the CUFE to optimal value in each EEZ, the total catch among the three areas (given by the harvest function H in Section 2.2) is 2.2 million tons/year, to be compared to the average landings of small pelagic fish in this area in 2005–2011 which ranges from ~ 1.5 to 2 million tons/year including several over-exploited species (FAO, 2012). Such over-exploitation rates may not be sustainable and increase the vulnerability of the ecosystem due to the effect of climate change. However, switching from the present over-fishing situation toward the maximum catch by acting on the cost parameters may cause the fishing effort and the landings to low on the short term.

Although the assumption we made were very different, we can compare our results with game theory models applied to study case of shared fisheries as for example the case of Canada, USA and Mexico (Javor et al., 2011; Lo et al., 2011), which are exploiting the same pacific sardine (*Sardinops sagax*) transbounday stock, with a status "declining/depleted" in 2011 (Based on US National Marine Fisheries Service). At a first glance, our results are in line with these studies (Ishimura et al., 2012, 2013). As Ishimura et al. (2012) we found that the cooperation between the countries leads to a higher total catch, but one of the countries (in our case the one with the highest carrying capacity density, zone 3) may get less landings when collaborating with other countries than if it minimises its fishing costs. Thus, the concerned country, in our case study Senegal, may have positive incentives to act non-cooperatively instead of participating to the cooperative management. Ishimura et al. (2013) demonstrated that this problem can be solved using side-payments, which are positive incentives given by the countries that may get benefit of cooperative management to the country that may have more interest to act as a free player i.e. non-cooperatively. Adapted to our case study it means that Zone 1 and 2 should provide a side payment to zone 3 to reduce their fishing efforts. Because the total catch increase in case of cooperative management, zone 1 and 2 would still increase their benefit from fisheries in comparison with the non-cooperative management situation. However, as explained in the previous paragraph, our dynamic model shows that the fishing agreements should be constantly tuned over time in order to maintain the system around the very particular case of optimal equilibrium. Also, taking into account the spatial variability of carrying capacity density inside the EEZ may change the results; in particular the very high concentration of fisheries observed off Cap Blanc in Mauritanian waters seems to indicate a particularly high carrying capacity in this area. Further refinement of the carrying capacity spatial and seasonal distribution will be needed for more informative management information.

3.1.2. Fully cooperative international fleet

In the case of the cooperative model, with the parameters listed in Table 1 and the cost per unit of fishing effort satisfies $B^* < K$, then there exists a single positive equilibrium (B^*, e^*). This equilibrium is stable ($\sigma > 0$, see Section 2.2), thus starting at any positive initial conditions, all trajectories are tending to this unique equilibrium (Fig. 5). This is a good situation because it allows maintaining a durable fishery in the long term. A desirable cooperative strategy would be also to avoid the case of a stable limit cycle because it would generate important fluctuations of the stock and of the fleets in the long term. For instance, at some periods of time, the fish stock would be very low, and this could lead to fish collapse as a result of any stochastic fluctuations. Such stochastic processes are not considered in our deterministic models but occur in the real world. As a consequence, the best cooperative strategy would be to maintain the fishery at the positive equilibrium (B^*, e^*) when it is globally asymptotically stable in the positive quadrant, i.e. when $B^* < K$ and $\sigma > 0$. Furthermore, it would also be needed to choose parameters in order to maintain the fishery at its Maximum Sustainable Yield (MSY) by fixing the costs at $c = c^*$. This can be achieved by the three nations upon a common agreement. Because we have several parameters that could be selected by fishery regulation, such as taxes, costs and maybe catchability (size net...), it would be possible to find a parameter set that maintains the fishery at its MSY. Such international fleet is difficult to implement in our case study as a split of objectives takes place at national level. Indeed the Senegalese need this fish resource for their food security rather than increase their GDP and Mauritania target the opposite. Nevertheless a smart agreement could target maximum sustainable yield goal providing in kind and cash contributions according to each national priority. The use of foreign distant fleet should be avoided to conciliate all parties seeking compromise from all sides. Finally, our result underline that the cooperative model is the optimal management scenario because the corresponding model is permanent. Indeed, there is a globally asymptotically stable equilibrium for the total fishery that can be reached by fixing CUFE in a rather large set of values. This contrasts with the competitive model for which it is needed to fix the CUFE in each EEZ at a single point in the whole parameter space.

3.2. Toward an operational virtual representation of transboundary fish stock

The present model may evolve toward an operational model for the management of shared fisheries. For that purpose, accurate data is needed about the nature of the fisheries in the different areas and the characteristics (species growth parameters, stock spatial distribution, and environmental carrying capacity) for the main fish populations exploited. This work stresses one more time that whatever the methodological approach, operational tool in fisheries management requests monitoring procedure of the biological and anthropic features of the system.

Compared to the gross estimate of the crude CUFE without integrating any tax or subsidy, (300 US$ per prospected km^2) and assuming that a similar fishing fleet operates in the three zones, setting the local CUFE according to the optimal case suggested by the model (Table 3) would implicate the implementation of additional tax on fisheries in zone 3 and 2 (optimal CUFE of ~ 400 and ~ 360 US$/km^2), but, oppositely, subsidies for fisheries in zone 1 (optimal CUFE of ~ 260 US$/km^2). However, actually the local fisheries in the three zones are of very different nature, which implies different crude CUFE. Indeed, the fishing cost in Senegal (zone 1) may be lower than in Mauritania (zone 2) if we assume that small scale fisheries (canoe) get lower cost than industrial fishing (vessel), which appears as an acceptable assumption. Small scale fisheries also operate in Mauritania waters but starting often from Senegal and thus get higher cost than in their home zone (Senegal). Moreover in Mauritania there is more industrial fishing activity (foreign fleet) than in Senegal where industrial fishing is very low (<5 vessels, CRODT, 2013). In Southern Morocco/Western Sahara most of the small pelagic fish comes from semi industrial fishing as found in Laayoune the main fishing harbour for small pelagic landing in Morocco (FAO, 2012). Thus, according to these qualitative criteria the actual crude CUFE should be ranked as follows: the lower cost may occur in Senegal, (mainly local fisheries, dominated by local small scale canoes), intermediate costs may occur in Morocco/Western Sahara (local fisheries, semi industrial boats), and the highest cost may occur in Mauritania (fisheries almost all foreign, mixed small scale and industrial fleets). According to our results, such CUFE ranking may exacerbate the disequilibrium already found assuming equal CUFE among the three zones (Fig. 2a), thus increasing the advantage of zone 3. Nevertheless, the average CUFE in Senegal may be increased if we also take into account the distant water fisheries DWF (Sumaila and Vasconcellos, 2000) and IUU (illegal, unreported and unregulated), which

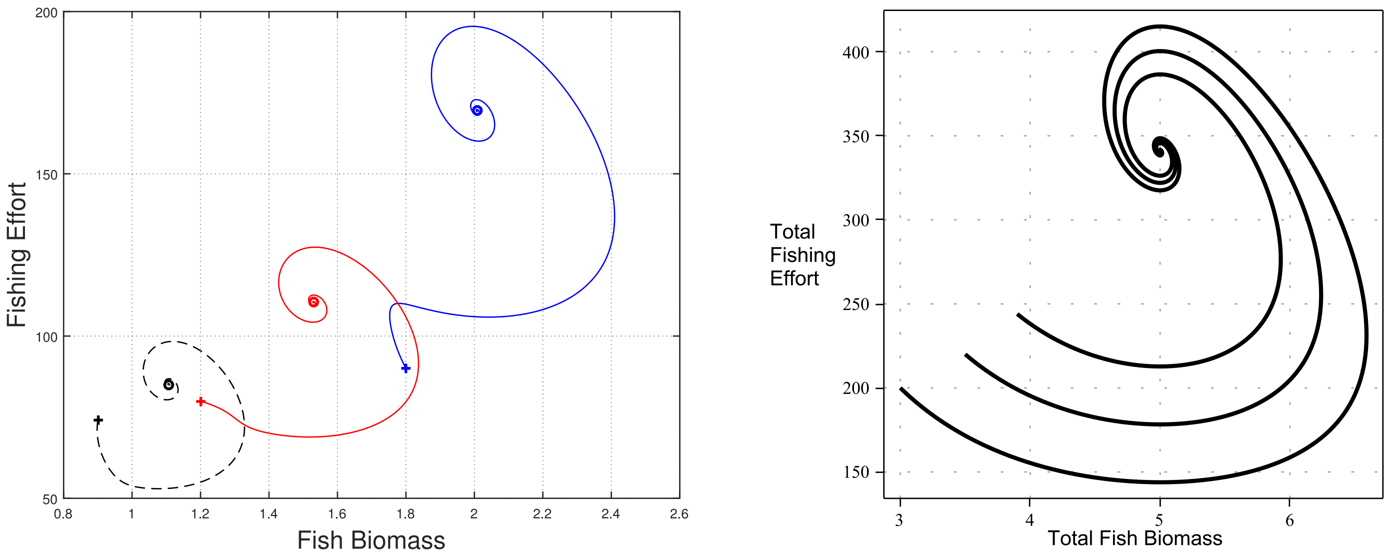

Fig. 5. Numerical simulations of the complete cooperative model, phase portraits with parameters values are in Table 1. Coexistence of the 3 fisheries when the costs per unit of effort among the three zones remain uniform. (a) The costs per unit of effort are similar to Fig. 2a: $c_1 = c_2 = c_3 = 300$. Blue, red and dotted lines correspond with the orbit of zone 1, 2 and 3, respectively using the complete model (5). Initial values are $B_1 = 1.8,\ B_2 = 1.2,\ B_3 = 0.9,\ e_1 = 90,\ e_2 = 80,\ e_3 = 74$. (b): Total fishing effort and total fish biomass of three zones, using the aggregated model (6) and the optimal cost per unit of effort $c_1 = c_2 = c_3 = c^* = 322.7449099$. The aggregated model has a single positive equilibrium which is globally asymptotically stable for different initial values. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

probably occur more in zone 3 than in the two other zones, due to low regulation and monitoring capabilities (Agnew et al., 2009). Finally, a precise estimate of the CUFE according to each fishery is necessary before setting eventual taxes or subsidies on the fishing effort to be implemented to tend toward optimal management. Within a given zone, such policy must be adjusted to each fishery type according to its crude CUFE.

As in all models dealing with fisheries issues, the hypotheses on the ecological properties of the exploited species are simplified to implement the model. Among these hypotheses we can report, the gravity centre of the spatial distribution of different exploited fish species may differ, as well as their growth rate and carrying capacity. Also, the results may be sensible to the existence of hot spots of fish density within a given zone, as for example the Cap Blanc area in Mauritania (Braham et al., 2014). Further development of the methodology presented in our work could include a realistic distribution of the carrying capacity within each zone (i.e. non uniform) based on physical and bio-geochemical simulations of the environment (e.g. Auger et al., 2016). The time dependent carrying capacity provided by the simulations of the environment could be included in the approach by using non-autonomous models.

The model might be easily generalized to the case of more than three countries sharing the same fish stock. Indeed, the Gambia could get different political goal than Senegal, Guinea-Bissau could be included in the process (Fig. 2) even if at the opposite Regional Fisheries Organisation (RFO) can simplify interactions i.e. in our case the Sub Regional Fisheries Commission (SRFC) could harmonize the rules for Mauritania, Senegal, The Gambia and even Guinea-Bissau.

A tricky point that happens in a large part of the world, especially in the third world, is the presence of illegal and unreported fishing activity that crosses the borders without fishing agreement (known as illegal, unreported, and unregulated (IUU) Fishing (Agnew et al., 2009)), following the fish migration or displacement. At a first glance, in our model this may cause the extinction of fisheries in zone 1 and 2. This might impact the efficiency of the fishing management proposed here and its equitability among the country. One main perspective of the work will be to study the impact of the presence of IUU acting among different countries where local fisheries are exploiting the same migratory fish population, taking as example the case of fisheries targeting sardinella in North–West Africa.

Finally, this approach provides a practical framework for building a reflection on the international management of shared exploited fish species *i.e.* a common issue, sometimes a matter of contention, for several RFO as international fisheries commissions, non governmental organisation (NGO) and national agencies. The model considers the fishing effort regulation through the control of the cost per unit of fishing effort (including tax or fishing permit), its analytical interpretation allows one to provide original fisheries management recommendations, which should encourage deeper studies for operational uses.

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Appendix A

Proof of Proposition 2.1. Put $\mathbb{R}^4_{+1} = \{x \in \mathbb{R}^4 : x_1 > 0 \text{ and } x_m \geq 0,\ m = \overline{2,4}\}$. At point A, we consider the Lyapunov function

$$V_A = p\left(B - K - K\log\frac{B}{K}\right) + \frac{c_1}{\phi}e_1 + \frac{c_2}{\phi}e_2 + \frac{c_3}{\phi}e_3.$$

Then, V_A is positive definite, radially unbounded in $\mathbb{R}^4_{+1}$ and

$$\dot{V}_A = -\frac{pr}{K}(B-K)^2 - \sum_{i=1}^{3}\left(c_i - \frac{Kpq\alpha_i}{S_i}\right)e_i = -\frac{pr}{K}(B-K)^2 - \sum_{i=1}^{3}\frac{pq\alpha_i}{S_i}(\overline{B}_i - K)e_i.$$

Since $\overline{B}_i \geq K$ for all $i=\overline{1,3}$ so $\dot{V}_A$ is negative semi-definite and the non-negative equilibrium points of system (2) only include *O, A*. By LaSalle's invariance principle, point *A* is globally asymptotically stable. Hence, $\lim_{t\to\infty} e_i(t) = 0,\ i=\overline{1,3}$. □

Proof of Theorem 2.2. *a)*: For fixed $i=\overline{1,3}$, correspond to P_i we consider the Lyapunov function

$$V_{P_i} = p\left(B - \overline{B}_i - \overline{B}_i\log\frac{B}{\overline{B}_i}\right) + \frac{c_i}{\phi}\left(e_i - \frac{rS_i(K-\overline{B}_i)}{Kq\alpha_i} - \frac{rS_i(K-\overline{B}_i)}{Kq\alpha_i}\log\frac{e_iKq\alpha_i}{rS_i(K-\overline{B}_i)}\right) + \frac{c_j}{\phi}e_j + \frac{c_k}{\phi}e_k$$

with *j, k* ≠ *i*. Then V_{P_i} is positive definite, radially unbounded in $\mathbb{R}^4_{+1}\cap\mathbb{R}^4_{+,i+1}$ and

$$\dot{V}_{P_i} = -\frac{pr}{K}(B-\overline{B}_i)^2 - \frac{pq\alpha_j}{S_j}(\overline{B}_j - \overline{B}_i)e_j - \frac{pq\alpha_k}{S_k}(\overline{B}_k - \overline{B}_i)e_k.$$

If $\overline{B}_i$ is the smallest then $\dot{V}_{P_i}$ is negative semi-definite on $\mathbb{R}^4_{+1}\cap\mathbb{R}^4_{+,i+1}$. Moreover, remaining equilibrium points will be unstable follow linearized principle. Therefore, by LaSalle's invariance principle then P_i is globally asymptotically stable. This yields to $\lim_{t\to\infty} e_k(t) = 0,\ k\neq i$.

b): If $e_i^* = 0$ or $e_j^* = 0$ then the equilibrium point is P_i, P_j in the case 1, respectively. With $e_i^*, e_j^* > 0$ we consider the Lyapunov function

$$V_{M_i} = p\left(B - \overline{B}_i - \overline{B}_i\log\frac{B}{\overline{B}_i}\right) + \frac{c_i}{\phi}\left(e_i - e_i^* - e_i^*\log\frac{e_i}{e_i^*}\right) + \frac{c_j}{\phi}\left(e_j - e_j^* - e_j^*\log\frac{e_j}{e_j^*}\right) + \frac{c_k}{\phi}e_k.$$

Then V_{M_i} is positive definite and

$$\dot{V}_{M_i} = -\frac{pr}{K}(B-\overline{B}_i)^2 - \frac{pq\alpha_k}{S_k}(\overline{B}_k - \overline{B}_i)e_k.$$

Argument as in proof of assertions (a), if $\overline{B}_k$ is the smallest then P_k is globally asymptotically stable and the equilibrium points in Γ_i are unstable. Otherwise, P_k is unstable and Γ_i is attraction set for all solution trajectories of the system. Thus, there exists *i* such that $\lim_{t\to\infty} e_i(t) = 0$.

c): If there exists $e_i^* = 0$ with $i=\overline{1,3}$ then the Lyapunov function of these type points is constructed as in the case 2. With $e_1^*, e_2^*, e_3^* > 0$ we consider then Lyapunov function

$$V_M = p\left(B - \overline{B}^* - \overline{B}^*\log\frac{B}{\overline{B}^*}\right) + \frac{c_1}{\phi}\left(e_1 - e_1^* - e_1^*\log\frac{e_1}{e_1^*}\right) + \frac{c_2}{\phi}\left(e_2 - e_2^* - e_2^*\log\frac{e_2}{e_2^*}\right) + \frac{c_3}{\phi}\left(e_3 - e_3^* - e_3^*\log\frac{e_3}{e_3^*}\right).$$

Then V_M is positive definite, radially unbounded in $\mathbb{R}^4_{++}$ and

$$\dot{V}_M = -\frac{pr}{K}(B-\overline{B}^*)^2 \leq 0.$$

By LaSalle's invariance principle, Γ is attraction set for all solution trajectories of the system. The next, we will show that solution of the system with initial condition $M_0(B_0, e_{10}, e_{20}, e_{30}) \in \mathbb{R}^4_{++}$ will converge to equilibrium point in $\Gamma\cap\mathbb{R}^4_{++}$. We consider an equilibrium point $M\in\Gamma\cap\mathbb{R}^4_{++}$, put $\Omega_c = \{x\in\mathbb{R}^4_+ : V_M(x)\leq c\}$. Then Ω_c is compact set, invariant with respect to the system (2) and $\Omega_c\subset\mathbb{R}^4_{++}$. Denote $E=\{x\in\Omega_c : \dot{V}_M(x) = 0\}$, then the biggest invariant set with respect to the system (2) in *E* is $L = E\cap\Gamma\subset\mathbb{R}^4_{++}$. By LaSalle's invariance principle, we conclude that every trajectory starting in Ω_c approaches *L*. Now, choice *c* is large enough such that $M_0\in\Omega_c$. □

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