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Fault Detection and Identification for a Class of Continuous Piecewise Affine Systems with Unknown Subsystems and Partitions

Nikolaos Moustakis[1], Bingyu Zhou[1,3], Thuan Le Quang[2], and Simone Baldi[1]

[1] *Delft Center for Systems and Control, Delft University of Technology, The Netherlands*
N.Moustakis@tudelft.nl, S.Baldi@tudelft.nl
[2] *Quy Nhon University, Department of Mathematics, Binh Dinh, Vietnam* lequangthuan@qnu.edu.vn
[3] *Siemens AG, Research in Energy and Electronics, Erlangen, Germany* bryan-by@hotmail.com

SUMMARY

This paper establishes a novel online fault detection and identification (FDI) strategy for a class of continuous piecewise affine systems (PWA), namely bimodal and trimodal PWA systems. The main contributions with respect to the state of the art are the recursive nature of the proposed scheme and the consideration of parametric uncertainties in both partitions and in subsystems parameters. In order to handle this situation, we recast the continuous PWA into its max-form representation and we exploit the recursive Newton-Gauss algorithm on a suitable cost function to derive the adaptive laws to estimate online the unknown subsystem parameters, the partitions and the loss in control authority for the PWA model. The effectiveness of the proposed methodology is verified via simulations applied to the benchmark example of a wheeled mobile robot. Copyright © 2011 John Wiley & Sons, Ltd.

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KEY WORDS: Piecewise affine unknown systems, fault detection and identification, online parameter estimation, unknown partitions.

1. INTRODUCTION

With the increased demand of reliability for control systems, much attention has been devoted by the control community in fault detection techniques for complex systems [1, 2, 3, 4]. Piecewise affine (PWA) systems constitute a special class of complex (in particular, hybrid) systems that has been extensively studied in the literature in many application domains: production control systems [5], robotics [6] and flight control systems [7], among others. A classical problem in the aforementioned application domains is the detection and identification of faults which might appear in the form of plant structural changes (usually associated to variations in the state matrix) or actuator faults (usually associated to changes in the input matrix). In the classical (non-hybrid) setting, the fault detection and identification (FDI) problem can be reformulated in terms of an estimation problem, i.e. it is assumed that faults in the system are reflected in a change of the parameters of the system model [8]. The situation with PWA systems is however more complex than classical estimation, because an extra uncertainty might occur: i.e. the partitions describing the switching from one mode to another might be uncertain or even change with time. Therefore, FDI of a PWA system involves the estimation of both the parameters of the submodels and the regions (hyperplanes)

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defining the partition of the state space. In other words, despite the more complex setting, also the fault detection and identification for PWA systems can be, in principle, reformulated as a parametric estimation problem. With reference to partitioning, two alternative assumptions can be distinguished: the partition is assumed known and fixed a priori, or the partition is unknown along with the unknown submodels. For the first case, estimation of the submodels can be carried out using standard linear identification techniques, therefore, no particular challenge appears. For the second case, both the subsystems and the partitions corresponding to each subsystem must be estimated. This issue implies a classification problem where each data point must be associated to the most suitable mode. Then, the regions are shaped to clusters of data where the strict relation among data classification, parameter estimation and region estimation makes the fault detection and identification problem hard to solve [9]. Despite the challenging task, there is a number of approaches in the literature of PWA systems dealing with this problem: the authors in [10] propose a statistical clustering approach to classify the data points and estimate the submodel parameters in order to reconstruct the polyhedral partition of the regressor domain. Further results dealing with the estimation problem include the bayesian statistical-based approach [11], the bounded-error procedure [12], the mixed-integer programming procedure [13]. A survey on further recent results for the estimation of PWA systems can be found in [14]. It has to be noted thought, that the vast majority of results for the estimation of PWA systems focuses on the development of estimation algorithms that work offline, i.e. from batches of data.

On the other hand, literature has provided also alternative sets of tools (non necessarily based on parameter estimation) for FDI in complex systems: a brief overview is given here. Model-based tools focusing on the detection and identification of the partial loss of control authority in PWA systems, frequently used to model actuator faults, are studied in [15, 16]. An observer-based fault estimation approach for discrete PWA systems is presented in [17], while [18] provides sufficient conditions in terms of linear matrix inequalities for the input-to-state stability of the estimator. A message passing algorithm for automatically propagating the effects of uncertainties in interconnected bilinear systems and derive probabilistic fault thresholds is proposed in [19]. In [20], a clustering approach based on the maximized expectation algorithm is used, and it is proven to identify effectively sudden or pre-existing faults into a hybrid, mixed discrete modes-continuous time states, setting. An online learning algorithm using a Lyapunov-based approach is carried out in [21], to prove robust fault detection for the case of multi input-multi output nonlinear systems. Estimation-based and observer-based FDI of PWA systems with parametric uncertainties, but with known partitions, is studied in [22] and [23], respectively. A map-based approach using parameter-estimation techniques is presented in [24], where the unknown parameters are estimated online and they are used to detect faults in the model. A dual estimation scheme is developed in [25] to detect parametric changes with partial state information. A comprehensive review presenting the state-of-the art FDI methods in the literature and their applications are given in [26, 27]: none of the aforementioned FDI approaches can deal with PWA systems with parametric uncertainties in both partitions and in subsystems parameters.

Closely related to FDI, special attention has been devoted to fault-tolerant controller (FTC) synthesis for complex systems, which aim to cope with the identification of partial loss of the control action and compensate for the later in the closed-loop hybrid or PWA systems. FTC architectures can be divided into two main categories; passive FTC methods which provide controller synthesis proven to guarantee stable performance both when the system works in nominal operation and under faulty conditions, and active FTC methods which are characterized by the reconfiguration of the controller when faulty conditions are detected [28]. An active FTC approach is adopted in [29]: set-valued observers detect faults by evaluating the inconsistency of input-output data and a multiple-model adaptive controller designed for the degraded system is connected to the loop with proven closed-loop stability. In [30], a fault-tolerant controller is proposed to guarantee stabilization and satisfactory system performance in case of partial loss of control authority in the control loop. A reconfigurable control approach for continuous PWA systems susceptible to actuator and sensor faults is given in [31]: by solving a set of linear matrix inequalities, this approach is proven robust to closed-loop stability and guarantees reference tracking. The authors in [32] investigate the adaptive

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fault-tolerant control problem in the presence of time-varying actuator faults. A robust control approach is carried out to prove asymptotic stability and robust performance in the case of combined actuator failures and disturbances. The adaptive FTC scheme in [33] is formulated to model actuator failures as Markovian jump systems subjected to stochastic noise. Then, a backstepping technique ensures boundedness of the closed-loop signals. A supervisor FTC scheme for the discrete event systems case is studied in [34]. The control goal of this work, is a desired, predetermined non-faulty behavior for the overall system, for any fault occurring within a bounded known delay. In [35], an intelligent-based (neural-network, fuzzy) FTC design with adaptive online estimation and control for linearized time-varying systems is introduced, and asymptotic tracking and uniform signals boundedness is evaluated under certain conditions on the system's dynamics. An overview of the diverse FTC schemes and their applications are given by [36, 37]: none of the aforementioned FTC approaches can deal with PWA systems with parametric uncertainties in both partitions and in subsystems parameters.

Therefore, to the best of the authors' knowledge, there is currently no online fault detection and identification technique developed for continuous PWA systems with joint unknown subsystems and partitions. The main contribution of this work is tackling, in a parameter estimation framework, the fault detection and identification problem for a class of continuous-time PWA systems, namely bimodal and trimodal continuous PWA systems, where the subsystems and the partition are jointly unknown. Without loss of generality, the unknown system partition is assumed to be generated by the so-called "centers", as defined in [38]. By exploiting this particular description, a novel parametric model is derived via the max-form of the PWA system, and consequently, a cost function depending on the estimation error is derived which is used to develop a recursive Gauss-Newton algorithm to obtain online the adaptive laws for all the parameters (i.e. the subsystem parameters and also the centers). It has to be noted that, differently from literature on estimation in PWA systems, the developed algorithm is completely online. *Online* FDI algorithms produce unknown system estimates at each time instant, by processing and evaluating the current measurements of signals. Because of this, they are commonly referred to as recursive FDI algorithms, to be distinguished from the *offline* or nonrecursive ones. For the latter case, also found in the literature as the batch FDI estimation algorithms, all signals' measurements are collected offline over large time interval horizons. In both online or offline methods, the unknown parameters are calculated by using optimization techniques on some appropriately chosen cost function. However, while parameter values estimated using online fault detection and parameter estimation architectures can vary with time as new data arrive, the parameters estimated using offline techniques do not (unless new batches of data are collected). Compared to the offline scheme, online recursive algorithms measure the system's signals continuously so as to update and correct the parameter estimates. Because they can update for fault occurrence incidents in the system and compensate for their resultant detrimental effect while the system is in operation, online estimation algorithms are conceptually superior for fault detection problems, as compared to the offline ones [39, 40].

The rest of this paper is organized as follows: Section 2 presents preliminaries for the PWA system representation and Sections 3 and 4 present the online fault detection and identification problem and the main result of this work, for bimodal and trimodal PWA systems respectively. The effectiveness of the online identification methodology is illustrated via simulations in Section 5. Finally, Section 6 concludes this paper summarizing the main findings and giving some recommendations for future work.

The notations used in this paper are standard:
$\mathbb{R}$: the set of real numbers;
$\mathbb{N}$: the set of nonnegative integers;
Given a vector $x = \begin{bmatrix} x_1 & x_2 & \cdots & x_m \end{bmatrix}^T \in \mathbb{R}^m$, the superscript T denotes its transpose and

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$$\mathrm{diag}\,(x) = \begin{bmatrix} x_1 & 0 & \cdots & 0 \\ 0 & x_2 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & x_m \end{bmatrix};$$

$\mathrm{r}_i(X)$: denotes the i^{th} row of matrix X.

2. PRELIMINARIES IN PWA SYSTEMS

We consider the bimodal PWA system of the form

$$\dot{x} = \begin{cases} A_1 x + B_1 \Lambda_1 u + e_1 & \text{if } (x,u) \in \mathcal{X}_1 \\ A_2 x + B_2 \Lambda_2 u + e_2 & \text{if } (x,u) \in \mathcal{X}_2 \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $B_i \in \mathbb{R}^{n\times m}$ are known matrices, $A_i \in \mathbb{R}^{n\times n}$, $e_i \in \mathbb{R}^n, i \in \{1,2\}$ are unknown matrices, $\Lambda_i \in \mathbb{R}^{m\times m}$ are unknown diagonal matrices. The term $B_i\Lambda_i$ models partial loss of control authority, and $\{\mathcal{X}_1, \mathcal{X}_2\}$ are polyhedral partitions of the state-input space. We take the regions $\mathcal{X}_1, \mathcal{X}_2$ are polyhedral partitions in to $\mathbb{R}^{n+m}$ (the state-input space), generated by the centers as defined in [38]. In fact, for general PWA systems (non necessarily bimodal), given $N \in \mathbb{N}, N \geqslant 2$ vectors $c_1, c_2, ..., c_N \in \mathbb{R}^{n+m}$ representing the centers, for each point $z = \begin{bmatrix} x, u \end{bmatrix}^T \in R^{n+m}$ in the state-input space, the polyhedral regions are defined as

$$\begin{aligned} \mathcal{X}_j &= \left\{ z \in \mathbb{R}^{n+m} \mid \left\| z - c_j \right\|_2 \leq \| z - c_k \|_2 \right\},\ k \neq j \\ &= \left\{ z \in \mathbb{R}^{n+m} \,| \mathcal{A}_j z \leq q_j \right\} \end{aligned} \tag{2}$$

where

$$\begin{aligned} \mathcal{A}_j &= 2 \begin{bmatrix} c_1 - c_j & c_2 - c_j & \cdots & c_N - c_j \end{bmatrix}^T, \\ q_j &= \begin{bmatrix} \beta_{1,j} & \beta_{2,j} & \cdots & \beta_{N,j} \end{bmatrix}, \end{aligned}$$

with $\beta_{k,j} = c_k^T c_k - c_j^T c_j$ for $j = 1, 2, ..., N$. For bimodal PWA systems with partitions $\mathcal{X}_1$ and $\mathcal{X}_2$ we have only two centers, c_1 and c_2 from (2). The regions $\mathcal{X}_1$, $\mathcal{X}_2$ are given by the following relations:

$$\mathcal{X}_1 = \left\{ (x,u) \mid 2(c_2 - c_1)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_2^T c_2 - c_1^T c_1) \leq 0 \right\} \tag{3a}$$

$$\mathcal{X}_2 = \left\{ (x,u) \mid 2(c_2 - c_1)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_2^T c_2 - c_1^T c_1) \geq 0 \right\} \tag{3b}$$

The system (1) is an extension in a PWA sense of classical uncertain systems used in adaptive and fault-tolerant control of multivariable linear systems [41, 42]. Fault detection and identification in classical uncertain systems can be performed by using parameter estimation techniques, e.g. by assuming that faults in the system are be reflected in a change of the (non-faulty) parameters in the system model [43]. A similar idea applies (albeit the more challenging task) to the PWA extension (1): the FDI problem then involves detecting any change in the system parameters of (1), as formulated in the following.

Problem 1
Derive a recursive (online) FDI algorithm with the capability of estimating the unknown system parameters, the unknown loss of control authority, and the unknown partitions of the PWA system (1). Also, embed in the FDI algorithm a finite-memory (or forgetting) mechanism so as to be able to detect (slowly) changes in the system parameters.

2.1. Max-form representation of bimodal PWA systems

It is assumed that the system (1) is continuous in the state space. By referring to [44], continuity of the system is equivalent to the existence and uniqueness of an $h \in \mathbb{R}^n$ such that

$$\begin{bmatrix} A_1 & B_1\Lambda_1 \end{bmatrix} - \begin{bmatrix} A_2 & B_2\Lambda_2 \end{bmatrix} = 2h(c_2 - c_1)^T \tag{4a}$$

$$e_1 - e_2 = -h(c_2^T c_2 - c_1^T c_1). \tag{4b}$$

In view of (4), system (1) can be written into its max-form representation as follows:

$$\dot{x} = \begin{bmatrix} A_2 & B_2\Lambda_2 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} + e_2 - h \max\left\{2(c_1 - c_2)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_1^T c_1 - c_2^T c_2), 0\right\}. \tag{5}$$

One can see that there are infinitely many pairs of centers (c_1, c_2) that can generate the polyhedral regions $\mathcal{X}_1$ and $\mathcal{X}_2$ in (3). However, if we fix one center to an arbitrary value, the other center is uniquely determined. Therefore, without loss of generality, we fix the center c_2 to be equal to a given value $\tilde{c}$ and we use the notations c, A, B, e, Λ in place of $c_1, A_2, B_2, e_2, \Lambda_2$, respectively. Then, (5) becomes

$$\dot{x} = Ax + B\,\mathrm{diag}\,(u)\lambda + e - h \max\left\{2(c - \tilde{c})^T \begin{bmatrix} x \\ u \end{bmatrix} - (c^T c - \tilde{c}^T \tilde{c}), 0\right\}, \tag{6}$$

where $\lambda \in \mathbb{R}^m$ in (6) is defined in vector form as $\lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_m \end{bmatrix}^T$, such that $\Lambda = \mathrm{diag}(\lambda)$.

Remark 1

Note that the clear benefit of (6) with respect to (1) is its economy with respect to parameters. In fact, in (1) we need to estimate $2(n^2 + m + n)$ parameters for the subsystems and $(n + m + 1)$ parameters for the partitions: on the other hand, in (6) we have $n^2 + m + n$ parameters for the subsystem $(A, B\Lambda, e)$ and $2n + m$ parameters for h and c. This is because (6) exploits explicitly the continuity of the PWA system.

3. ONLINE IDENTIFICATION OF BIMODAL PWA SYSTEMS

By following a FDI approach based on parameter estimation, as in [43], Problem 1 for the PWA system (6) can be recast to the minimization of the following cost function

$$J(t, \hat{\theta}) = \frac{1}{2} \int_0^t e^{-\xi(t-s)} \left\| x(s) - \hat{x}(s, \hat{\theta}) \right\|^2 ds$$

which can be component-wisely written as

$$J(t, \hat{\theta}) = \frac{1}{2} \int_0^t e^{-\xi(t-s)} \sum_{i=1}^{n} \left(\hat{x}_i(s, \hat{\theta}) - x_i(s)\right)^2 ds \tag{7}$$

where $\xi > 0$ corresponds to the forgetting factor which is a design parameter, θ denotes the unknown parameter which contains all the healthy (non-faulty) or faulty values of the parameters, which appear in the form of plant structural changes (associated to variations in the state matrix A and the affine vector e), actuator faults (associated to changes in the input vector λ), or mode partition faults (associated to changes in the vector h and the center c). In addition, after collecting the true parameters in

$$\theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_n \\ \lambda \\ c \end{bmatrix}, \text{ with } \theta_i = \begin{bmatrix} \mathrm{r}_i(A)^T \\ e_i \\ h_i \end{bmatrix} \text{ for } i = 1, 2, ..., n, \tag{8}$$

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where e_i and h_i in (8) are the scalar components of the vectors e and h, we have that $\hat{\theta}$ are the estimated values of θ computed by the minimization of (7). The state $\hat{x}(s, \hat{\theta})$ is the observed state for the system (6), which is computed through the following Luenberger-like observer

$$\begin{aligned} \dot{\hat{x}}(s, \hat{\theta}) = A_m \hat{x}(s, \hat{\theta}) + (\hat{A} - A_m) x(s) + B \operatorname{diag}(u(s)) \hat{\lambda} + \hat{e} \\ - \hat{h} \max \left\{ \Psi(\hat{c}, x(s), u(s)), 0 \right\} \end{aligned} \tag{9}$$

where $\Psi(\hat{c}, x(s), u(s)) = 2(\hat{c} - \tilde{c})^T \begin{bmatrix} x(s) \\ u(s) \end{bmatrix} - (\hat{c}^T \hat{c} - \tilde{c}^T \tilde{c})$ and A_m is a Hurwitz matrix. The Luenberger-like observer (9) is an extension in PWA sense of the parallel-series estimator used for classical linear systems [45]. The solution of (9) can be calculated explicitly as follows:

$$\begin{aligned} \hat{x}(s, \hat{\theta}) = e^{A_m} x_0 + \int_0^s e^{A_m(s-\tau)} \Bigg\{ \\ \begin{bmatrix} [x^T 1 \, -\max\{\Psi, 0\}] \cdots & 0 \\ 0 \quad \ddots & 0 \\ 0 & \cdots [x^T 1 \, -\max\{\Psi, 0\}] \end{bmatrix} \begin{bmatrix} r_1(\hat{A} - A_m) \\ \hat{e}_1 \\ \hat{h}_1 \\ \vdots \\ r_n(\hat{A} - A_m) \\ \hat{e}_n \\ \hat{h}_n \end{bmatrix} \\ + B \operatorname{diag}(u) \hat{\lambda} \Bigg\} d\tau. \end{aligned} \tag{10}$$

The unknown parameter θ is estimated with the recursive Gauss-Newton algorithm. Then, $\hat{\theta}$ is updated online via the following adaptive law

$$\dot{\hat{\theta}}(t) = -\Gamma U(t)^{-1} \Phi(t) \begin{bmatrix} \frac{\partial J(t, \hat{\theta})}{\partial \hat{x}_1} \\ \vdots \\ \frac{\partial J(t, \hat{\theta})}{\partial \hat{x}_n} \end{bmatrix} \Bigg|_{\hat{\theta}(0) = \hat{\theta}_0} \tag{11}$$

where $\Gamma > 0$ is the adaptation gain decided by the designer and

$$\dot{U}(t) = -\xi U(t) + \Phi(t) \Phi(t)^T, \; U(0) = 0 \tag{12}$$

with

$$\Phi(t) = \begin{bmatrix} \frac{\partial \hat{x}_1(t, \hat{\theta})}{\partial \hat{\theta}_1} & \frac{\partial \hat{x}_2(t, \hat{\theta})}{\partial \hat{\theta}_1} & \cdots & \frac{\partial \hat{x}_n(t, \hat{\theta})}{\partial \hat{\theta}_1} \\ \frac{\partial \hat{x}_1(t, \hat{\theta})}{\partial \hat{\theta}_2} & \frac{\partial \hat{x}_2(t, \hat{\theta})}{\partial \hat{\theta}_2} & \cdots & \frac{\partial \hat{x}_n(t, \hat{\theta})}{\partial \hat{\theta}_2} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \hat{x}_1(t, \hat{\theta})}{\partial \hat{\lambda}} & \frac{\partial \hat{x}_2(t, \hat{\theta})}{\partial \hat{\lambda}} & \cdots & \frac{\partial \hat{x}_n(t, \hat{\theta})}{\partial \hat{\lambda}} \\ \frac{\partial \hat{x}_1(t, \hat{\theta})}{\partial \hat{c}} & \frac{\partial \hat{x}_2(t, \hat{\theta})}{\partial \hat{c}} & \cdots & \frac{\partial \hat{x}_n(t, \hat{\theta})}{\partial \hat{c}} \end{bmatrix}. \tag{13}$$

In order to calculate recursively all the terms in (13), one can see that $\hat{x}(t, \hat{\theta})$ can be written in the following form

$$\hat{x}(t, \hat{\theta}) = g_0(t) + g_1(t) \hat{\theta} + g_2 \hat{\lambda} \tag{14}$$

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with

$$g_0(t) = e^{A_m t}x_0 - A_m \int_0^t e^{A_m(t-\tau)}x(\tau)d\tau,$$

$$g_1(t)=\int_0^t e^{A_m(t-\tau)} \begin{bmatrix} [x(\tau)^T 1 - \max\{\Psi,0\}] \cdots & 0 \\ 0 \quad \ddots & 0 \\ 0 \quad \cdots & [x(\tau)^T 1 - \max\{\Psi,0\}] \end{bmatrix} d\tau,$$

$$g_2(t) = \int_0^t e^{A_m(t-\tau)}B \ \operatorname{diag}(u(\tau))d\tau.$$

where Ψ is intended as $\Psi(\hat{c}, x(\tau), u(\tau))$. By using (10), the following relations are true:

$$\frac{\partial \hat{x}(t,\hat{\theta})}{\partial \hat{\theta}} = g_1(t) \tag{15a}$$

$$\hat{x}(t,\hat{\theta}) - x(t) = g_0(t) - x(t) + g_1(t)\hat{\theta} + g_2(t)\hat{\lambda} \tag{15b}$$

$$\frac{\partial \hat{x}(t,\hat{\theta})}{\partial \hat{\lambda}} = g_2(t) \tag{15c}$$

and

$$\frac{\partial \hat{x}(t,\hat{\theta})}{\partial \hat{c}} = -\int_0^t e^{A_m(t-\tau)}\hat{h} \begin{bmatrix} w_1(\tau) \\ \vdots \\ w_{n+m}(\tau) \end{bmatrix}^T d\tau \tag{15d}$$

where

$$w_j(\tau) = \begin{cases} 2\mathrm{x}_j(\tau) - 2\hat{c}_j(\tau) & , \Psi(\hat{c},\tau) = \max\{\Psi(\hat{c},\tau), 0)\} \\ 0, & \text{, otherwise} \end{cases} \tag{16}$$

for $j = 1, 2, ..., n$, where

$$\mathrm{x}_j(\tau) = \begin{cases} x_j(\tau) & , j = 1, 2, ..., n \\ u_{j-n}(\tau) & , j = n+1, ..., n+m. \end{cases}$$

From (7) and (15b) it can be proven

$$\frac{d}{dt}\Big(\frac{\partial J(t,\hat{\theta})}{\partial \hat{x}}\Big) = -\xi \frac{\partial J(t,\hat{\theta})}{\partial \hat{x}} + g_0(t) - x(t) + g_1(t)\hat{\theta}(t) + g_2(t)\hat{\lambda}(t) \tag{17}$$

and because of (13), (15a), (15c), (15d), relation (13) is equivalently represented by

$$\Phi(t) = \begin{bmatrix} g_1^T(t) \\ g_2^T(t) \\ \frac{\partial \hat{x}(t)}{\partial \hat{c}}^T \end{bmatrix}. \tag{18}$$

To update g_0, g_1, g_2 and $\frac{\partial \hat{x}(t)}{\partial \hat{c}}$ we use the fact that

$$\dot{g_0} = A_m g_0 - A_m x, \ g_0(0) = x(0) \tag{19a}$$

$$\dot{g_1} = A_m g_1 + \begin{bmatrix} [x^T 1 - \max\{\Psi,0\}] \cdots & 0 \\ 0 \quad \ddots & 0 \\ 0 \quad \cdots & [x^T 1 - \max\{\Psi,0\}] \end{bmatrix}, \quad g_1(0) = 0 \tag{19b}$$

$$\dot{g_2} = A_m g_2 + B \ \operatorname{diag}(u), \ g_2(0) = 0. \tag{19c}$$

$$\frac{d}{dt}\Big(\frac{\partial \hat{x}}{\partial \hat{c}}\Big) = A_m \frac{\partial \hat{x}}{\partial \hat{c}} - \hat{h}\begin{bmatrix} w_1 \\ \vdots \\ w_{n+m} \end{bmatrix}^T, \quad \frac{\partial \hat{x}}{\partial \hat{c}}(0) = 0 \tag{19d}$$

with $w_1, w_2, ..., w_{n+m}$ defined in (16). The recursive design is complete and the local optimality of the resulting fault detection and identification method for PWA systems is remarked hereafter.

Remark 2
Because (1) is nonlinear with respect to the estimated parameters, the cost function (17) is nonconvex with respect to $\hat{\theta}$, even after the max-form representation (6). As a consequence, a global optimum minimizing the cost function (17) cannot be guaranteed for every initial condition (even in the presence of persistency of excitation). In other words, only convergence to local optima can be guaranteed in general: therefore, the Gauss-Newton algorithm will exhibit best performance when the initial estimate $\hat{\theta}_0$ lies in a small neighborhood of θ. To the best of the authors' knowledge, there is no estimation method for PWA systems with joint unknown subsystems and partitions that can guarantee global optimality.

Remark 3
Note that in case the partitions $\{\mathcal{X}_1, \mathcal{X}_2\}$ are known, the parameter c is given, and (6) results in a linear-in-the-parameter model for which standard converge results apply [46], after a slight revision of the proposed method in order to get rid of $\frac{\partial \hat{x}(t)}{\partial \hat{c}}$.

4. ONLINE IDENTIFICATION OF TRIMODAL PWA SYSTEMS

The proposed framework can be extended to trimodal continuous PWA systems with minor modifications. Similarly to the bimodal PWA system case studied in Section 2, the trimodal PWA system reads as

$$\dot{x} = \begin{cases} A_1 x + B_1 \Lambda_1 u + e_1 & \text{if } (x,u) \in \mathcal{X}_1 \\ A_2 x + B_2 \Lambda_2 u + e_2 & \text{if } (x,u) \in \mathcal{X}_2 \\ A_3 x + B_3 \Lambda_3 u + e_3 & \text{if } (x,u) \in \mathcal{X}_3 \end{cases} \tag{20}$$

where

$$\begin{aligned}
\mathcal{X}_1 = \Big\{(x,u) \mid\, & 2(c_2 - c_1)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_2^T c_2 - c_1^T c_1) \leq 0, \\
& 2(c_3 - c_1)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_3^T c_3 - c_1^T c_1) \leq 0\Big\}, \\
\mathcal{X}_2 = \Big\{(x,u) \mid\, & 2(c_2 - c_1)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_2^T c_2 - c_1^T c_1) \geq 0, \\
& 2(c_3 - c_2)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_3^T c_3 - c_2^T c_2) \leq 0\Big\}, \\
\mathcal{X}_3 = \Big\{(x,u) \mid\, & 2(c_3 - c_2)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_3^T c_3 - c_2^T c_2) \geq 0, \\
& 2(c_3 - c_1)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_3^T c_3 - c_1^T c_1) \geq 0\Big\}.
\end{aligned}$$

4.1. Max-form representation of trimodal PWA systems

In order to write the max-form presentation of the PWA system in (20), one has to distinguish between two cases:

Case 1: The centers c_1, c_2, c_3 lie on a line. Without loss of generality, it is assumed that the center c_2 lies on the segment $[c_1, c_3]$. Similarly to the bimodal PWA system case, the continuity of

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the PWA system (20) is equivalent to the existence and uniqueness of $h_1, h_2 \in \mathbb{R}^n$ such that (20) can be equivalently written as

$$\begin{aligned}\dot{x} = &\begin{bmatrix} A_2 & B_2\Lambda_2 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} + e_2 \\ &- h_1 \max\left\{2(c_2 - c_1)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_2^T c_2 - c_1^T c_1), 0\right\} \\ &- h_3 \max\left\{2(c_1 - c_3)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_1^T c_1 - c_3^T c_3), 0\right\}.\end{aligned} \tag{22}$$

Case 2: The centers c_1, c_2, c_3 do not lie on a line. The continuity of (20) is equivalent to the existence and uniqueness of $h_1, h_2, h_3 \in \mathbb{R}^n$ such that

$$\begin{aligned}\begin{bmatrix} A_1 & B_1\Lambda_1 \end{bmatrix} - \begin{bmatrix} A_2 & B_2\Lambda_2 \end{bmatrix} &= 2h_1(c_2 - c_1)^T, \\ e_1 - e_2 &= -h_1(c_2^T c_2 - c_1^T c_1),\end{aligned} \tag{23a}$$

$$\begin{aligned}\begin{bmatrix} A_2 & B_2\Lambda_2 \end{bmatrix} - \begin{bmatrix} A_3 & B_3\Lambda_3 \end{bmatrix} &= 2h_2(c_3 - c_2)^T, \\ e_2 - e_3 &= -h_2(c_3^T c_3 - c_2^T c_2),\end{aligned} \tag{23b}$$

$$\begin{aligned}\begin{bmatrix} A_3 & B_3\Lambda_3 \end{bmatrix} - \begin{bmatrix} A_1 & B_1\Lambda_1 \end{bmatrix} &= 2h_3(c_3 - c_1)^T, \\ e_3 - e_1 &= -h_3(c_3^T c_3 - c_1^T c_1).\end{aligned} \tag{23c}$$

Lemma 1
For the vectors h_1, h_2, h_3 in (23), it is true

$$h_1 = h_2 = -h_3. \tag{24}$$

Proof
Relation (23) gives

$$(h_3 + h_2)c_3^T + (h_1 - h_2)c_2^T - (h_1 + h_3)c_1^T = 0. \tag{25}$$

If c_1, c_2, c_3 are linearly independent, it follows from (25) that $h_1 = h_2 = -h_3$. For the case c_1, c_2, c_3 are linearly dependent, one center can be written as a linear combination of the two other centers. Without loss of generality, let $c_3 = \alpha c_1 + \beta c_2$, with $\alpha, \beta \in \mathbb{R}$ such that $\alpha + \beta \neq 1$. It follows that

$$\begin{aligned}(h_3 + h_2)c_3^T &= \alpha(h_3 + h_2)c_1^T + \beta(h_3 + h_2)c_2^T, \\ (h_3 + h_2)c_3^T &= (h_1 + h_3)c_1^T + (h_2 - h_1)c_2^T,\end{aligned} \tag{26}$$

implying $h_2 + h_3 = (\alpha + \beta)(h_2 + h_3)$, and hence $h_2 = -h_3$. Substituting this result in (25), it follows $h_1 = h_2$ and the lemma is proved.

□

In view of (23) and Lemma 1, if we define $h_1 = h_2 = -h_3 = h$, then the PWA system (20) is given in its max-form presentation as

$$\begin{aligned}\dot{x} = &\begin{bmatrix} A_3 & B_3\Lambda_3 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} + e_3 - h \max\left\{2(c_2 - c_3)^T \begin{bmatrix} x \\ u \end{bmatrix}\right. \\ &\left. - (c_2^T c_2 - c_3^T c_3), 2(c_1 - c_3)^T \begin{bmatrix} x \\ u \end{bmatrix} - (c_1^T c_1 - c_3^T c_3), 0\right\}.\end{aligned} \tag{27}$$

Remark 4
As demonstrated from the above discussion, the max-form presentation of the trimodal PWA system in (20) can have two different forms, (22) or (27), depending on whether the centers lie on a line or not. Once the appropriate max-form is determined, the adaptive update laws are developed in similar fashion as in the bimodal PWA system case.

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5. SIMULATION RESULTS

5.1. Bimodal PWA system

In this section we evaluate the effectiveness of the online fault detection and identification technique on the wheeled mobile robot (WMR) shown in Fig. 1, and presented in [47].

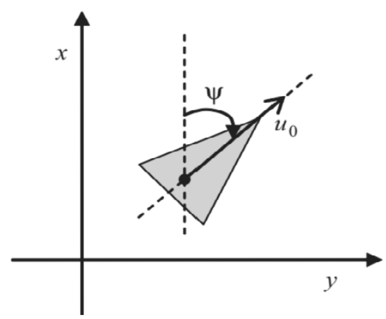

Figure 1. Schematic representation of the Wheeled Mobile Robot (WMR) [47]

The WMR is assumed to be rigid and it is driven by a torque T to control the heading angle ψ. The forward velocity of the robot, u_0, is in the direction of the $X-$body axis and is assumed to be constant, by designing appropriately a cruise controller. The heading angle of the WMR ψ is measured with respect to the positive X-axis in the inertial frame. The kinematic equations for the WMR are

$$\begin{aligned} \dot{y} &= u_0 \sin(\psi) \\ \dot{\psi} &= R \end{aligned} \tag{28}$$

and the dynamic equation of the WMR is

$$\dot{R} = 0.75 \frac{1}{I} T \tag{29}$$

where T is the input to the system, corresponding to the torque generated by the DC motors, 0.75 is the unknown actuator effectiveness, and $I = 1\ kg \cdot m^2$ (which is known) corresponds to the moment of inertia of the WMR with respect to the center of its mass. Inspired by this example, we consider as the actual system the bimodal PWA system in the form (1), with

$$A_1 = \begin{bmatrix} 0 & \frac{2}{\pi}u_0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & -\frac{2}{\pi}u_0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 0 & 0 & \frac{1}{I} \end{bmatrix}^T,\ \Lambda_1 = \Lambda_2 = 0.75.$$

$$e_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ e_2 = \begin{bmatrix} 2u_0 \\ 0 \\ 0 \end{bmatrix},$$

with $u_0 = 1$ which is unknown. The matrices above arise from approximating, in the range $\left[-\pi/2, 3\pi/2\right]$, the sinusoid with two straight lines (one straight line passes though the origin with slope $2/\pi$, while the other one passes though the point $(\pi, 0)$ with slope $-2/\pi$). As a consequence, the switching surface between the two subsystems is given by

$$\begin{bmatrix} 0 & \frac{2}{\pi} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ u \end{bmatrix} - 1 \leq 0,\ (\geq 0) \tag{30}$$

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where $[x_1 \ \ x_2 \ \ x_3 \ \ u] = [y \ \ \psi \ \ R \ \ T]$. The surface can be equivalently expressed by the two centers c_1, c_2, defined as follows:

$$c_1 = \begin{bmatrix} 0.25 & \frac{\pi}{2} - 0.25 & 0.25 & 0.25 \end{bmatrix}^T,$$
$$c_2 = \begin{bmatrix} 0.25 & \frac{\pi}{2} + 0.25 & 0.25 & 0.25 \end{bmatrix}^T.$$

Note that the definition of c_1 and c_2 is not unique: however, by fixing c_2, the other center c_1 would be uniquely determined. We acknowledge that, in this particular example, the partitions might be known: however, to be consistent with our setting and illustrate the proposed method, we assume that the partitions are unknown.

In view of the structure of the matrices, only five parameters are unknown and need to be determined: the nonzero term in the first row of A_2, the nonzero term in e_2 (representing uncertainties or changes in the cruise speed), the scalar term Λ_2 (representing uncertainties or changes in the actuator effectiveness), the unique nonzero term in h, and the second entry of c_1 (representing uncertainties in the partition). Therefore, by defining θ properly, it is possible to use a priori knowledge of the matrix structure and derive a Gauss-Newton method that estimates only the relevant five parameters (details are not shown for compactness). The design parameters have been taken as:

$$A_m = \begin{bmatrix} 0 & -0.637 & 0 \\ 0 & 0 & 1 \\ 0.003 & -0.054 & -0.114 \end{bmatrix}, \ \xi = 0.5, \ \Gamma = \mathrm{diag}\,(0.01, 0.03, 0.85, 0.03, 0.01)$$

where the eigenvalues of A_m are stable (one real eigenvalue and one complex conjugate pair). The initial state is taken as $x_0 = [1 \ \ \pi/2 \ \ 0]^T$. In order to provide enough persistency of excitation, the input is a series of steering and counter-steering sinusoids at frequency 0.2, 0.8 and $1.6 \ rad/s$.

In order to check consistency of the approach we have selected many $\hat{\theta}(0)$ randomly (zero mean Gaussian noise with covariance 0.1) in a neighborhood of θ. For all initial conditions the convergence was consistent, and Figs. 2 and 3 show one simulation. In addition, Figs. 4 and 5 show the capability to track some (slow) variation in time of the parameters: these variations have been simulated by slightly increasing u_0 and decreasing the actuator effectiveness).

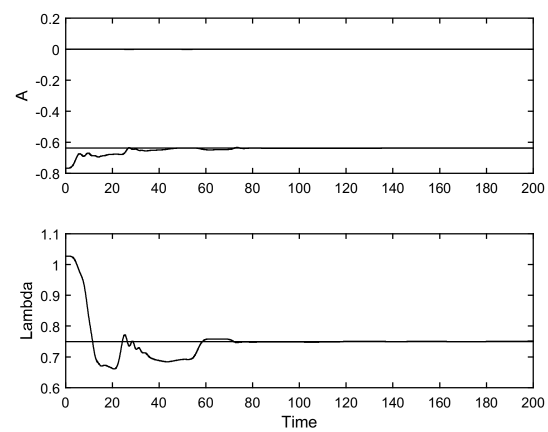

Figure 2. Online identification of A_2 and Λ_2 when c_2 is known (the true parameter values are shown in red color lines)

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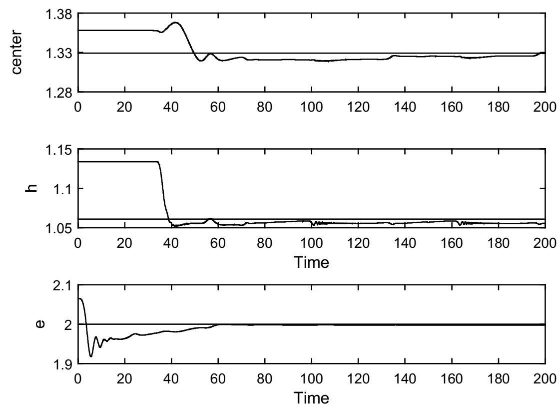

Figure 3. Online identification of e_2, h and c_1 when c_2 is known (the true parameter values are shown in red color lines)

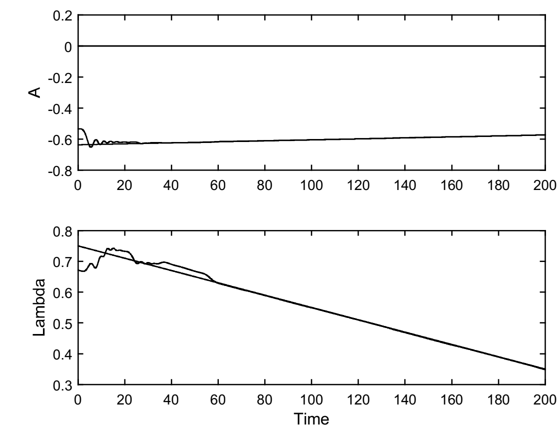

Figure 4. Online identification of A_2 and Λ_2 when c_2 is known for slow variations (the true parameter values are shown in red color lines)

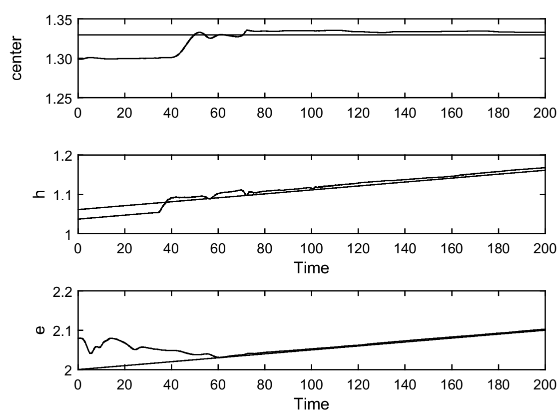

Figure 5. Online identification of e_2, h and c_1 when c_2 is known for slow variations (the true parameter values are shown in red color lines)

Remark 5
In order to highlight nonlinearity of the problem and the possibility of getting trapped into local minima, Table I shows the distance between the true and the estimated parameters (at steady-state) $\left\| \theta - \hat{\theta}_{st} \right\| / \|\theta\|$, as a function of the variance of $\theta - \hat{\theta}(0)$. The table highlight that when the initial condition is very far from the true parameter, the steady state distance also increases: this happens because the Gauss-Newton algorithm may not converge to the actual parameters.

$Var\left[\theta - \hat{\theta}(0)\right]$	$Avg\left[\frac{\|\theta-\hat{\theta}_{st}\|}{\|\theta\|}\right]$
0.03	0.2%
0.1	0.4%
0.3	0.8%
1.0	4.2 %
3.0	18.8 %

Table I. Performance depending on the initial estimate

5.2. Trimodal PWA system

In order to show the effectiveness of the proposed approach also in a trimodal setting, we take the example from [48] . This example has all the centers on a line, and notice that e_1 and e_3 have been modified with respect to [48] so as to make the PWA system continuous. In particular, we have

$$
\begin{aligned}
A_1 &= \begin{bmatrix} 0 & 1 \\ -1.5 & -1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}, \quad e_1 = \begin{bmatrix} 0 \\ 1.4 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}, \\
A_3 &= \begin{bmatrix} 0 & 1 \\ -2.5 & -1 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 0 \\ 1.4 \end{bmatrix}
\end{aligned}
$$

and $\Lambda_1 = \Lambda_2 = \Lambda_3 = 0.75$. The switching surface is defined in terms of the three centers

$$
\begin{aligned}
c_1 &= \begin{bmatrix} -4 & 0 & 0 \end{bmatrix}^T, \\
c_2 &= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T, \\
c_3 &= \begin{bmatrix} 4 & 0 & 0 \end{bmatrix}^T
\end{aligned}
$$

By exploiting a similar form as in (22), we formulate the FDI problem as the one of estimating the parameters of A_2, Λ_2, e_2, the vectors h_1 and h_3, and the centers c_1 and c_3 (we assume that the center c_2 is known). We have used $x_0 = [0.5 \ \ -0.5]^T$, a multi-sinusoid input (with 3 sinusoids), and the design parameters

$$
A_m = \begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix}, \ \xi = 0.05, \ \Gamma = \mathrm{diag}\,(1, 1, 1, 1, 1, 1, 1, 0.05, 0.05, 40, 40)
$$

where the zero components of h_1, h_3, c_1 and c_3 are not estimated. The results from the proposed online FDI algorithm are given in Fig. 6 (for A_2 and Λ_2), Fig. 7 (for e_2, h_1 and h_3), and Fig. 8 (for c_1 and c_3). It is observed that all estimates converge to the correct values after some transient.

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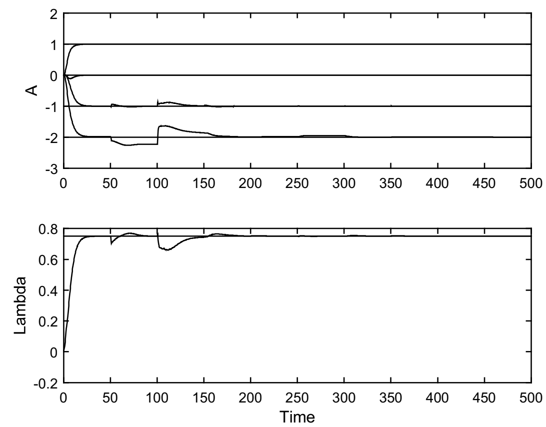

Figure 6. Online identification of A_2 and Λ_2 when c_2 is known (the true parameter values are shown in red color lines)

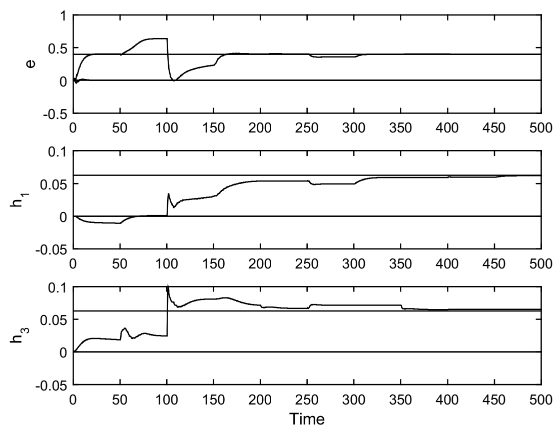

Figure 7. Online identification of e_2, h_1 and h_3 when c_2 is known (the true parameter values are shown in red color lines)

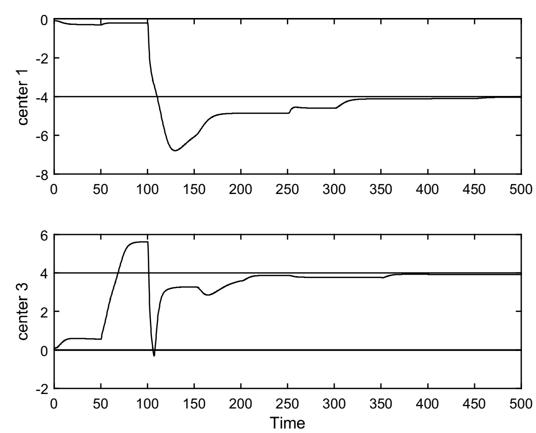

Figure 8. Online identification of c_1 and c_3 when c_2 is known (the true parameter values are shown in red color lines)

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6. CONCLUSION

This paper has established a novel online fault detection and identification strategy for a class of continuous piecewise affine systems (PWA), namely bimodal and trimodal PWA systems. The approach is estimation-based, i.e. it is assumed that faults in the system are reflected in a change of the parameters of the system model. The main contributions with respect to the state of the art are the recursive nature of the proposed scheme and the consideration of parametric uncertainties in both partitions and in subsystems parameters. In order to handle this situation, we recast the continuous PWA into its max-form representation and we exploited the recursive Newton-Gauss algorithm on a suitable cost function to derive the adaptive laws to estimate online the unknown subsystem parameters, the partitions and the loss in control authority for the PWA model. The effectiveness of the proposed methodology was verified via simulations applied to the benchmark example of a wheeled mobile robot. Future work could include the extension beyond trimodal systems: a possible idea to deal with this situation is to have multiple bimodal or trimodal estimator and a switching logic, according to architectures as in [49].

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