VIASM-IAMP Summer School in Mathematical Physics, Quy Nhon 2023

Titles & Abstracts

Minicourse: Nonlinear Waves in fluids and general relativity

John R. L. Anderson

In these lectures, I will describe some problems which are of current interest in the study of hyperbolic pdes. I will focus on a few simple examples where we prove either global stability or singularity formation. A common theme we will explore is first understanding things in symmetry and only afterwards upgrading our understanding of the phenomena to a more general setting. The main tools I will introduce for going outside of symmetry are various energy estimates in the context of the commuting vector field method. Emphasis will be placed on an intuitive understanding of the properties of wave propagation which allow us to prove the results. Moreover, these results will be motivated in the physical settings of general relativity and gas dynamics. Finally, I hope to describe some of the relationships between the topics we cover with other areas of current interest.

Reference: Lecture notes by Jonathan Luk on wave equations would be useful:
https://web.stanford.edu/~jluk/NWnotes.pdf

Minicourse: Some aspects of stochastic quantization

Nguyen Viet Dang

In this mini course, I will try to introduce some of the main ideas of stochastic quantizations through several examples. We shall mostly focus on the stochastic Allen-Cahn equation in 2 dimensional space. I will outline how to derive the Gibbs measures of the celebrated Ginzburg-Landau models in the plane as the stationary measure of the stochastic PDE.

Minicourse: Statistical mechanics of Coulomb and Riesz gases

Mathieu Lewin

Coulomb and Riesz gases play a very important role in Physics. Typical examples include galaxies or self-gravitating stars, charged systems such as plasmas. But long range Riesz gases also appear in many unexpected mathematical situations, including Ginzburg-Landau vortices, eigenvalues of random matrices, zeros of the Riemann function, modular forms and sphere packing problems.

In this course, I will review what is known and unknown for Coulomb and Riesz gases, from the point of view of statistical mechanics. I will start by recalling what has been proved in the short range case, before turning to more complicated long range systems including Coulomb.

Reference: The lecture will be based on a recent review on the subject, published by the Journal of Mathematical Physics: http://dx.doi.org/10.1063/5.0086835.

Minicourse: Analysis of topological defects in nematic liquid crystals

Luc Nguyen

Description: In these lectures, we discuss recent progresses and challenges in the study of defects in mathematical theories of nematic liquid crystals. We will start with a brief survey of topological point defects for vector-valued maps in Oseen-Frank and Ginzburg-Landau theories. We then move on with tensor-valued maps in Landau-de Gennes theory, which have been studied more intensively recently. The lectures will focus on aspects related to minimality, stability, uniqueness, and symmetry of stationary solutions.

Public Lecture: Is Mathematics Unreasonably Effective? Why?

Sergiu Klainerman

In his famous essay "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" Eugene Wigner describes the mysterious nature of the relationship between Mathematics and Natural Sciences, especially Physics, and provides various striking examples. I will discuss these examples and more and attempt to analyze the roots of this remarkable effectiveness.

Workshop lecture: Stable submanifolds in the product of projective spaces

Maria Alejandra Ramirez Luna

If Σ_i is a stable submanifold of M_i, for i = 1, 2, then $\Sigma_1 \times \Sigma_2$ is a stable submanifold in $M_1\times M_2$. Are all the stable submanifolds in $M_1\times M_2$ like that? We will show that this is the case for specific dimensions and codimensions in the product of a complex or quaternionic projective space with any other Riemannian manifold. We will also talk about the behavior of stable submanifolds under a complex structure of the product of two complex projective spaces. We will describe how our proofs were motivated by work that has been done by Simons, Lawson, Ohnita, Torralbo and Urbano. Part of this work is joint with Shuli Chen (Stanford).

Workshop lecture: Random Schrödinger operators with complex decaying potentials

Konstantin Merz

We estimate complex eigenvalues of continuum random Schrödinger operators of Anderson type. Our analysis relies on methods of J. Bourgain (Discrete Contin. Dyn. Syst., 2002, Lecture Notes in Math., 2003) related to almost sure scattering for random lattice Schrödinger operators, and allows us to consider potentials which decay almost twice as slowly as in the deterministic case. The talk is based on joint work with Jean-Claude Cuenin.

Workshop lecture: 1D focusing Bose gases with two- and (critical) attractive three-body interactions

Dinh-Thi Nguyen

We consider a one-dimensional, trapped, focusing Bose gas where N bosons interact with each other via both a scaled two-body interaction potential and an attractive scaled three-body interaction potential. The system is stable if and only if the three-body interaction is not too negative. In the mean-field regime, we prove that the many-body system exhibits the Bose--Einstein condensation on the cubic-quintic NLS ground states. In the collapse regime, we prove that the ground state of the system is fully condensed on the (unique) solution to the quintic NLS equation. We also discuss the case of critical three-body interaction.

Workshop lecture: Asymptotics of Helmholtz-Kirchhoff Point-Vortices in the Phase Space

Trinh T. Nguyen

In the 1858-1876, Helmholtz-Kirchhoff introduced the point-vortex system, which describes the vortex dynamics for ideal fluid flows on the whole plane. In 1872, Boltzmann derived his famous equations, which describe the statistical behavior of gas particles in elastic binary collisions. The Hilbert 6th problem aims to establish a connection between the mesoscopic Boltzmann equations and the macroscopic fluid models in formal limits. We give a precise pointwise estimate for the Boltzmann solutions by the Hilbert-type expansion around the Euler singular point-vortex system. This is joint work with Chanwoo Kim (UW-Madison).

Workshop lecture: A new point of view on the Einstein constraint equations with arbitrary mean curvature

Quoc Anh Ngo

Lying at the center of mathematical physics for several decades, the Einstein constraint equations have already been studied intensively since an effective version of the conformal method first appeared in the early 1970s. However, only until the Yamabe problem was solved, the existence and uniqueness of solutions to the constraints were known but limited to the constant mean curvature (CMC) case. This was in the early 90s. Analogous results for the near-CMC case began to appear thereafter. In the last twenty years, there has been some limited progress toward the understanding of solutions to the constraints in the far-from-CMC case. Although it was initially conceivable that these far-from-CMC results would lead to a new picture for the non-CMC case that would mirror the good properties of the CMC and near-CMC cases, remarkable examples of bifurcations, of non-existence, and of non-uniqueness of solutions have been discovered. In this talk, I will present a simple approach to constructing solutions to the constraints in certain far-from-CMC regimes. This new approach implies that some known existence results on Yamabe positive manifolds with arbitrary mean curvature can be thought of as perturbations of the CMC case. This is based on joint work with Romain Gicquaud.

Workshop lecture: Fractional maximal distributions (FMDs) and application to regularity theory

Minh-Phuong Tran

For the purpose of acquiring a good understanding of spaces of functions whose modulus to a power p is integrable, such as Lebesgue or Lorentz spaces, we discuss the definition of distribution functions. The knowledge of distribution function provides sufficient information to evaluate the norm of a function in these spaces. From this key idea, we present the notion of “fractional maximal distribution functions" (FMDs) by the joint with fractional maximal operators. As an application of the approach with FMDs, we establish the regularity theory for nonlinear equations of different types.
