The prescribed Ricci curvature problem

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Abstract: We will begin by introducing the background from Riemannian geometry required to formulate the prescribed Ricci curvature problem. After that, we will state the problem itself and place it into the context of modern geometric analysis. The last two lectures will survey some of the classical results and outline the main directions of research on the topic.

Lecture plan:

Lecture 1. Manifolds, diffeomorphisms, Riemannian metrics, covariant derivatives.

Lecture 2. Curvature: Riemannian, Ricci, scalar. Motivation and intuition for the Ricci curvature. Diffeomorphism invariance and the Bianchi identity.

Lecture 3. Equations with Ricci curvature: Einstein, Ricci flow, prescribed curvature equation. Poincare conjecture and connections with physics.

Lecture 4. Prescribed curvature problems: scalar, Ricci, Riemannian. The prescribed Ricci curvature problem as a system of PDEs. Non-ellipticity. DeTurck's local existence theorem and inverse function theorem techniques.

Lecture 5. Using symmetries to solve the prescribed Ricci curvature problem. Solvability on SU(2) and other homogeneous spaces. Cohomogeneity one symmetries.

References:

Classic Riemannian geometry books: Lee, Do Carmo, Gallot–Hulin–Lafontaine, Jost. Besse book: introduction and Chapter 5. Also:

R.S. Hamilton, The Ricci curvature equation, in: Seminar on nonlinear partial differential equations (S.-S. Chern, ed.), Springer-Verlag, New York, 1984, 47–72.

T. Buttsworth, A. Pulemotov, The prescribed Ricci curvature problem for homogeneous metrics, in: Differential geometry in the large (O. Dearricott et al., eds), Cambridge University Press, 2021, 169–192.