THE LOCALLY $\mathcal{F}$-APPROXIMATION PROPERTY OF BOUNDED HYPERCONVEX DOMAINS

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Abstract. In this paper, we study the local property of bounded hyperconvex domains Ω which we can approximative each plurisubharmonic function $u \in \mathcal{F}(\Omega)$ by an increasing sequence of plurisubharmonic functions defined on strictly larger domains.

1. Introduction

Hed [10] give in 2012 the following definition of the $\mathcal{F}$-approximation property of bounded hyperconvex domains.

Definition 1.1. A bounded hyperconvex domain Ω in $\mathbb{C}^n$ has the $\mathcal{F}$-approximation property if there exists a sequence of hyperconvex domains $\{\Omega_j\}$ such that $\Omega \Subset \Omega_{j+1} \Subset \Omega_j$ and we can approximate each function $u \in \mathcal{F}(\Omega)$ by an increasing sequence of functions $u_j \in \mathcal{F}(\Omega_j)$ quasi everywhere on Ω.

The first result in this direction is the theorem of Benelkourchi [2] in 2006 about the approximation of plurisubharmonic functions. Cegrell and Hed [6] proved in 2008 that a sufficient condition for Ω to have the $\mathcal{F}$-approximation property is that one single function in the class $\mathcal{N}(\Omega)$ can be approximated with functions in $\mathcal{N}(\Omega_j)$. Hed [9] proved in 2010 that if Ω has the $\mathcal{F}$-approximation property then we can approximate each function with given boundary values $u \in \mathcal{F}(\Omega, f|_\Omega)$ by an increasing sequence of functions $u_j \in \mathcal{F}(\Omega_j, f|_{\Omega_j})$ a.e. on Ω. Later, Benelkourchi [3] studied in 2011 the approximation of plurisubharmonic functions in the weighted energy class. Amal [1] studied in 2014 the approximation of plurisubharmonic functions in the weighted energy class with given boundary values. Recently, Hong [11] proved in 2015 a generalization of Cegrell and Hed's theorem.

The purpose of this paper is to study the local property of the $\mathcal{F}$-approximation property. Namely, we prove the following theorem.

Theorem 1.2. *Let $\Omega \Subset \Omega_{j+1} \Subset \Omega_j$ be bounded hyperconvex domains in $\mathbb{C}^n$ such that $\overline{\Omega} = \bigcap_{j=1}^{\infty} \Omega_j$. Then Ω has the $\mathcal{F}$-approximation property if only if Ω has the locally $\mathcal{F}$-approximation property, i. e., for every $z \in \partial\Omega$ there exists a neighborhood U_z of z such that $\Omega \cap U_z$ has the $\mathcal{F}$-approximation property.*

This result is proved using the $\mathcal{F}$-plurisubharmonic functions and the technique of Coltoiu and Mihalache [7].

The organization of the paper is as follows. In Section 2 we recall some notions of pluripotential theory which is necessary for the next results of the paper. In Section 3 we prove the main result of the paper.

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2. Preliminaries

Some elements of pluripotential theory that will be used throughout the paper can be found in [1]-[15]. Let Ω be a domain in $\mathbb{C}^n$. We denote by $PSH(\Omega)$ ($PSH^-(\Omega)$) the family of plurisubharmonic (negative plurisubharmonic) functions.

2.1. Cegrell's classes
We recall some Cegrell's classes of plurisubharmonic functions. Let Ω be a bounded hyperconvex domain in $\mathbb{C}^n$, i.e. a connected, bounded open subset of $\mathbb{C}^n$ such that there exists a negative plurisubharmonic function ρ such that $\{z \in \Omega : \rho(z) < -c\} \Subset \Omega$, $\forall c > 0$. Put

$$\mathcal{E}_0(\Omega) = \left\{\varphi \in PSH^-(\Omega) \cap L^\infty(\Omega) : \lim_{z\to\partial\Omega} \varphi(z) = 0,\ \int_\Omega (dd^c\varphi)^n < \infty\right\},$$

$$\mathcal{F}(\Omega) = \left\{\varphi \in PSH^-(\Omega) : \exists \mathcal{E}_0 \ni \varphi_j \searrow \varphi, \sup_j \int_\Omega (dd^c\varphi_j)^n < \infty\right\}$$

and

$$\mathcal{E}(\Omega) = \big\{\varphi \in PSH^-(\Omega) : \forall G \Subset \Omega, \exists u_G \in \mathcal{F}(\Omega),\ u = u_G \text{ on } G\big\}.$$

Let $\varphi \in \mathcal{E}(\Omega)$ and let $\{\Omega_j\}$ a fundamental sequence of Ω, i.e, Ω_j be strictly pseudoconvex domains such that $\Omega_j \Subset \Omega_{j+1} \Subset \Omega$ and $\bigcup_{j=1}^{\infty} \Omega_j = \Omega$. Put

$$\varphi^j = \sup\{u \in PSH(\Omega) : u \leqslant \varphi \text{ on } \Omega\backslash\Omega_j\}$$

and

$$\mathcal{N}(\Omega) = \{\varphi \in \mathcal{E}(\Omega) : \varphi^j \nearrow 0 \text{ a. e. in } \Omega\}.$$

2.2. The plurifine topology
The plurifine topology $\mathcal{F}$ on open subsets of $\mathbb{C}^n$ is the weakest topology in which all plurisubharmonic functions are continuous. Notions pertaining to the plurifine topology are indicated with the prefix $\mathcal{F}$ and notions pertaining to the fine topology are indicated with $\mathbb{C}^n$. For a set $A \subsetneq \mathbb{C}^n$ we write $\overline{A}$ for the closure of A in the one point compactification of $\mathbb{C}^n$, $\overline{A}^{\mathcal{F}}$ for the $\mathcal{F}$-closure of A and $\partial_{\mathcal{F}}A$ for the $\mathcal{F}$-boundary of A. We denote by $\mathcal{F}$-$PSH(\Omega)$ the set of $\mathcal{F}$-plurisubharmonic functions on an $\mathcal{F}$-open set Ω.

Note that if Ω be an open subsets of $\mathbb{C}^n$ then $\mathcal{F}$-$PSH(\Omega) = PSH(\Omega)$.

3. Proof of Theorem 1.2

First, we need the following auxiliary result. The idea of the proof is to use the $\mathcal{F}$-plurisubharmonic functions.

Lemma 3.1. *Let $\Omega \subset \mathbb{C}^n$ be bounded hyperconvex domains. Assume that there exists a sequence of bounded hyperconvex domains $\{\Omega_j\}$ such that $\Omega \Subset \Omega_{j+1} \Subset \Omega_j$ and $\overline{\Omega} = \bigcap_{j=1}^\infty \Omega_j$. Then the following statements are equivalent.*

(a) if $u \in \mathcal{E}_0(\Omega)$ and define $u_j := \sup\{\varphi \in PSH^-(\Omega_j) : \varphi \leqslant u \text{ in } \Omega\}$ then $1_{\Omega_j}u_j$ converges uniformly to $1_\Omega u$ in $\mathbb{C}^n$.

(b) there exists $u_j \in PSH^-(\Omega_j)$ such that $(\sup_j u_j)^* \in \mathcal{N}(\Omega)$.

(c) there exists $u \in \mathcal{N}(\Omega)$, $u_j \in PSH^-(\Omega_j)$ such that $u_j \to u$ a. e. in Ω.

(d) Ω has the $\mathcal{F}$-approximation property.

Proof. (a) $\Rightarrow$ (b) $\Rightarrow$ (c) is obvious. (c) $\Rightarrow$ (d): see [6]. We prove (d) $\Rightarrow$ (a). Let $u \in \mathcal{E}_0(\Omega)$. Since Ω has the $\mathcal{F}$-approximation property so there exists a sequence of hyperconvex domains $\{U_j\}$ and sequence of functions $\psi_j \in \mathcal{F}(U_j)$ such that $\Omega \Subset U_{j+1} \Subset U_j$ and $\psi_j \nearrow u$ a. e. in Ω. Without loss of generality we can assume that $\Omega_j \subset U_j$. Put

$$u_j := \sup\{\varphi \in PSH^-(\Omega_j) : \varphi \leqslant u \text{ in } \Omega\}.$$

It is clear that $u_j \in \mathcal{E}_0(\Omega_j)$ and $u_j \leqslant u_{j+1}$ in Ω_{j+1}. We claim that u_j is maximal plurisubharmonic function in a open neighborhood of $\Omega_j \backslash \Omega$. Indeed, put $\delta = \sup_{\Omega_{j+1}} u_j$. Since $\Omega_{j+1} \Subset \Omega_j$ and $u_j \in \mathcal{E}_0(\Omega_j)$ so $\delta < 0$. Put

$$G_j := \Omega_j \backslash (\Omega \cap \overline{\{u < \delta/2\}}).$$

Since $\overline{\{u < \delta/2\}} \Subset \Omega$ so G_j be a open neighborhood of $\Omega_j \backslash \Omega$. Since $\{u > \delta/2\} \cap \Omega \subset \{u_j < u\} \cap \Omega$ so from Theorem 1.1 in [11] we have $(dd^c u_j)^n = 0$ in G_j. Hence, u_j is maximal plurisubharmonic function in G_j. This proves the claim.

Since $\psi_j \leqslant u_j \leqslant u$ in Ω so $u_j \nearrow u$ a.e. in Ω. Choose $\psi \in \mathcal{F}(\Omega)$ such that $u_j \nearrow u$ in $\Omega \backslash \{\psi = -\infty\}$. Put $\Omega' := \Omega \backslash \{\psi = -\infty\}$. Let $k \in \mathbb{N}^*$. Since $\{u \leqslant -\frac{1}{k}\} \Subset \Omega$ and

$$\{u_j \leqslant -\frac{1}{k}\} \cap \Omega' \searrow \{u \leqslant -\frac{1}{k}\} \cap \Omega'$$

as $j \nearrow +\infty$ so there exists an increasing sequence $\{j_k\}$ such that $\{u_{j_k} \leqslant -\frac{1}{k}\} \cap \Omega' \Subset \Omega$ for all k. By replacing $\{u_j\}$ with its subsequence if necessary, we can assume that

$$\{u_j \leqslant -\frac{1}{j}\} \cap \Omega' \Subset \Omega$$

for every $j \geqslant 1$. Put

$$v_j = \begin{cases} u_j & \text{in } \{u_j \geqslant -\frac{1}{j}\} \cap \Omega' \\ \max(u_j, u - \frac{1}{j}) & \text{in } \{u_j < -\frac{1}{j}\} \cap \Omega'. \end{cases}$$

Since $u - \frac{1}{j} < -\frac{1}{j} = u_j$ in $\{u_j = -\frac{1}{j}\}$ so by Proposition 2.3 in [13] we have v_j is $\mathcal{F}$-plurisubharmonic function in Ω'. Since $\{\psi = -\infty\}$ is pluripolar and $\mathcal{F}$-closed in Ω so by Theorem 3.7 in [12] the function

$$v_j^*(z) := \mathcal{F}\text{-}\limsup_{\Omega' \ni \zeta \to z} v_j(\zeta),\ z \in \Omega$$

is $\mathcal{F}$-plurisubharmonic function in Ω. Since Ω be open subset of $\mathbb{C}^n$ so from Proposition 2.14 in [12] we have $v_j^* \in PSH^-(\Omega)$.

We claim that $u_j = v_j^*$ in Ω. Indeed, since $\{\psi = -\infty\}$ is a pluripolar subset of Ω and $u_j = v_j$ in $\Omega \backslash (\overline{\{u_j < -\frac{1}{j}\} \cap \Omega'})$ so $u_j = v_j^*$ in $\Omega \backslash (\overline{\{u_j < -\frac{1}{j}\} \cap \Omega'})$. Put

$$\varphi = \begin{cases} v_j^* & \text{in } \Omega \\ u_j & \text{in } \Omega_j \backslash \Omega. \end{cases}$$

Then, $\varphi \in PSH^-(\Omega_j)$ and $\varphi \leqslant u$ in Ω. Hence, $\varphi \leqslant u_j$ in Ω_j. Moreover, since $\varphi = v_j^* \geqslant u_j$ in Ω so $u_j = v_j^*$ in Ω. This proves the claim. Since $u - \frac{1}{j} \leqslant v_j \leqslant u$ in Ω' so $u - \frac{1}{j} \leqslant u_j \leqslant u$ in Ω. Moreover, since u_j is maximal plurisubharmonic function in a open neighborhood of $\Omega_j \backslash \Omega$ and $u_j \geqslant -\frac{1}{j}$ in $\partial(\Omega_j \backslash \Omega)$ so $u_j \geqslant -\frac{1}{j}$ in $\Omega_j \backslash \Omega$. Therefore,

$$1_\Omega u - \frac{1}{j} \leqslant 1_{\Omega_j} u_j \leqslant 1_\Omega u$$

in $\mathbb{C}^n$. Hence, $1_{\Omega_j} u_j$ converges uniformly to $1_\Omega u$ in $\mathbb{C}^n$. The proof is complete. □

Remark 3.2. Let $\Omega \subset \Omega_{j+1} \subset \Omega_j$ be bounded open subsets of $\mathbb{C}^n$ such that Ω has the $\mathcal{F}$-approximation property and $\bigcap_{j=1}^{\infty} \Omega_j \subset \overline{\Omega}$. If $u \in \mathcal{E}_0(\Omega)$ and

$$u_j := \sup\{\varphi \in PSH^-(\Omega_j) : \varphi \leqslant u \text{ in } \Omega\}$$

then $1_{\Omega_j} u_j$ converges uniformly to $1_\Omega u$ in $\mathbb{C}^n$. Indeed, since Ω has the $\mathcal{F}$-approximation property so there exists a sequence of hyperconvex domains $\{U_j\}$ such that $\Omega \Subset U_{j+1} \Subset U_j$ and $\bigcap_{j=1}^{\infty} U_j = \overline{\Omega}$. Without loss of generality we can assume that $\Omega_j \subset U_j$. Put

$$v_j := \sup\{\varphi \in PSH^-(U_j) : \varphi \leqslant u \text{ in } \Omega\}.$$

Since $v_j \leqslant u_j$ in Ω_j so $1_{U_j} v_j \leqslant 1_{\Omega_j} u_j \leqslant 1_\Omega u$ in $\mathbb{C}^n$. By Lemma 3.1 we have $1_{U_j} v_j$ converges uniformly to $1_\Omega u$ in $\mathbb{C}^n$. Hence, $1_{\Omega_j} u_j$ converges uniformly to $1_\Omega u$ in $\mathbb{C}^n$.

We now give the proof of theorem 1.2. The idea of the proof is taken from [7] (also see [8], [15]).

Proof of theorem 1.2. The necessity is obvious. We prove the sufficiency. Let $U_j'' \Subset U_j' \Subset U_j$, $j = 1, \ldots, m$ are open subsets such that $U_j \cap \Omega$ has the $\mathcal{F}$-approximation property and $\partial\Omega \Subset \bigcup_{j=1}^{m} U_j''$. Without loss of generality we can assume that $\Omega_1 \backslash \Omega \Subset \bigcup_{j=1}^{m} U_j''$. Let $u^j \in \mathcal{E}_0(\Omega \cap U_j)$ and define

$$u_k^j = \sup\{\varphi \in PSH^-(\Omega_k \cap U_j) : \varphi \leqslant u^j \text{ in } \Omega \cap U_j\}.$$

Without loss of generality we can assume that $-1 \leqslant u_k^j \leqslant 0$ for all $j = 1, \ldots, m$ and for any $k \in \mathbb{N}^*$. From the proof of Theorem 1 in [7] (also see the proof of Proposition 3.2 in [8]) there exists a convex continuous increasing function $\tau : (-\infty, 0) \to (0, +\infty)$ and a positive number $\varepsilon_0 \in (0, 1)$ such that $\lim_{x \to 0} \tau(x) = +\infty$ and

$$|\tau(u^j - \varepsilon) - \tau(u^k - \varepsilon)| \leqslant 1 \text{ in } U_j \cap U_k \cap \Omega$$

for all $k, j = 1, \ldots, m$ and for any $\varepsilon \in (0, \varepsilon_0)$. Let $\{\varepsilon_j\} \subset (0, \varepsilon_0)$ such that $\varepsilon_j \searrow 0$. Since τ is continuous function so there exists a decreasing sequence of positive real numbers $\{\delta_j\}$ such that $\delta_j \searrow 0$ and

$$\tau(x - \varepsilon_j) - \tau(x - \varepsilon_j - \delta) \leqslant \min\left(\frac{\tau(-\varepsilon_j - \delta_j - 1)}{j}, 1\right)$$

for any $x \in [-1, 0]$, for any $\delta \in (0, \delta_j]$. By Remark 3.2 we have $1_{\Omega_k \cap U_j} u_k^j$ converges uniformly to $1_{\Omega \cap U_j} u^j$ in $\mathbb{C}^n$. Hence, by replacing $\{u_k^j\}$ with a subsequence if necessary, we can assume that

$$1_{\Omega \cap U_j} u^j - \delta_k \leqslant 1_{\Omega_k \cap U_j} u_k^j \leqslant 1_{\Omega \cap U_j} u^j$$

in $\mathbb{C}^n$. Therefore,

$$|\tau(u_h^j - \varepsilon_h) - \tau(u_h^k - \varepsilon_h)| \leqslant 3$$

in $U_j \cap U_k \cap \Omega_h$ for any $k, j = 1, \ldots, m$. Choose $\chi_j \in \mathcal{C}_0^\infty(\mathbb{C}^n)$ satisfying $0 \leqslant \chi_j \leqslant 1$, $\text{supp}\chi_j \Subset U_j'$ and $\chi_j = 1$ on a neighborhood of U_j''. Let $A > 0$ so large that $|z|^2 - A < 0$ on Ω_1 and that $\chi_j(z) + A|z|^2$ is plurisubharmonic in $\mathbb{C}^n$ for every $j = 1, \ldots, m$. Put

$$v_h^j(z) = \tau(u_h^j(z) - \varepsilon_h) + 3(\chi_j(z) + A|z|^2 - A^2 - 1), \ z \in \Omega_h \cap U_j$$

and

$$v_h(z) = \max\left\{\frac{v_h^j(z)}{\tau(\varepsilon_h)} - 1 : z \in U_j'\right\}.$$

Since $v_h^j \in PSH(\Omega_h \cap U_j)$ and $v_h^j \leqslant v_h^k$ in $\partial U_j' \cap U_k'' \cap \Omega_h$ so v_h is a negative plurisubharmonic function in $\Omega_h \cap (\bigcup_{j=1}^m U_j'')$. Put $\Omega' = \Omega \cap (\bigcup_{j=1}^m U_j'')$ and define

$$v = \left(\sup_{h\geqslant 1} v_h\right)^*$$

in Ω'. Then $v \in PSH(\Omega')$. We claim that $v < 0$ in Ω'. Indeed, let $G \Subset \Omega'$ be an open set. Choose $\delta > 0$ such that $U_j' \cap G \subset \{u^j < -\delta\} \cap U_j'$ for any $j = 1, \ldots, m$. Since $u_h^j \leqslant u^j$ in $\Omega \cap U_j$ so

$$\begin{aligned} v_h(z) &\leqslant \max\left\{\frac{\tau(u^j(z) - \varepsilon_h)}{\tau(-\varepsilon_h)} - 1 : z \in \mathbb{B}_j'\right\} \\ &\leqslant \frac{\tau(-\delta - \varepsilon_h)}{\tau(-\varepsilon_h)} - 1 \end{aligned}$$

for all $z \in G$. Hence, $v < 0$ in G. This proves the claim. Let $K \Subset \Omega$ be an open subset of Ω such that $\partial K \Subset \Omega'$ and $\Omega \backslash K \subset \Omega'$. Put $B = \sup_{\partial K} v < 0$ and define

$$w = \begin{cases} B & \text{in } K \\ \max(v, B) & \text{in } \Omega\backslash K. \end{cases}$$

Then $w \in PSH^-(\Omega)$. We claim that $w \in \mathcal{N}(\Omega)$. Indeed, let $\varepsilon > 0$. Choose $h \in \mathbb{N}^*$ such that $\frac{3(A^2+1)}{\tau(-\varepsilon_h)} < \frac{\varepsilon}{2}$ and $\left(1 + \frac{1}{h}\right)\left(1 - \frac{\varepsilon}{2}\right) < 1$. Choose $\varepsilon_h' > \varepsilon_h$ such that $\left(1 + \frac{1}{h}\right)\left(1 - \frac{\varepsilon}{2}\right)\tau(-\varepsilon_h) < \tau(-\varepsilon_h')$. Then, we have

$$\begin{aligned} \{w < -\varepsilon\} \cap \Omega &\subset (\{v < -\varepsilon\} \cap \Omega') \cup K \\ &\subset (\{v_h < -\varepsilon\} \cap \Omega') \cup K \\ &\subset \bigcup_{j=1}^m \left(\left\{\frac{v_h^j}{\tau(-\varepsilon_h)} - 1 < -\varepsilon\right\} \cap \Omega \cap U_j\right) \cup K \\ &\subset \bigcup_{j=1}^m \left(\left\{\frac{\tau(u_h^j - \varepsilon_h) - 3(A^2+1)}{\tau(-\varepsilon_h)} < 1 - \varepsilon\right\} \cap \Omega \cap U_j\right) \cup K \\ &\subset \bigcup_{j=1}^m \left(\left\{\frac{\tau(u_h^j - \varepsilon_h)}{\tau(-\varepsilon_h)} < 1 - \frac{\varepsilon}{2}\right\} \cap \Omega \cap U_j\right) \cup K. \end{aligned}$$

Since

$$\tau(x - \varepsilon_h) \leqslant \tau(x - \varepsilon_h - \delta) + \frac{\tau(-\varepsilon_h - \delta_h - 1)}{h} \leqslant \left(1 + \frac{1}{h}\right)\tau(x - \varepsilon_h - \delta)$$

for all $x \in [-1, 0]$, for any $\delta \in (0, \delta_h]$ so

$$\left(1 + \frac{1}{h}\right)\tau(u_h^j - \varepsilon_h) \geqslant \tau(u^j - \varepsilon_h)$$

in $\Omega \cap U_j$. Hence,

$$\begin{aligned} \{w < -\varepsilon\} \cap \Omega &\subset \bigcup_{j=1}^m \left(\left\{\frac{\tau(u^j - \varepsilon_h)}{\tau(-\varepsilon_h)} < \left(1 + \frac{1}{h}\right)\left(1 - \frac{\varepsilon}{2}\right)\right\} \cap \Omega \cap U_j\right) \cup K \\ &\subset \bigcup_{j=1}^m (\{\tau(u^j - \varepsilon_h) < \tau(-\varepsilon_h')\} \cap \Omega \cap U_j) \cup K \end{aligned}$$

$$\subset \bigcup_{j=1}^{m} (\{u^j < \varepsilon_h - \varepsilon'_h\} \cap \Omega \cap U_j) \cup K.$$

Since $\{u^j < \varepsilon_h - \varepsilon'_h\} \cap \Omega \cap U_j \Subset \Omega$ for all $j = 1, \ldots, m$ so $\{w < -\varepsilon\} \cap \Omega \Subset \Omega$. It follows that $w \in \mathcal{N}(\Omega)$. This proves the claim. Now put

$$w_j = \begin{cases} B & \text{in } K \\ \max(v_j, B) & \text{in } \Omega_j \backslash K. \end{cases}$$

Then, $w_j \in PSH^-(\Omega_j)$ and $(\sup_j w_j)^* = w \in \mathcal{N}(\Omega)$. Hence, by Lemma 3.1 we get Ω has the $\mathcal{F}$-approximation property. The proof is complete. □

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