Univariate times series

In practice

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Table of contents

Table of contents

Box and Jenkins methodology

1. stationarity (if needed),
2. order selection,
3. estimation,
4. diagnostic checking,
5. final order selection,
6. forecasting,
7. ex post analysis.

Table of contents

Methods

- trend-seasonality decomposition,
- differencing transformations,
- Box-Cox transformation.

Differencing transformations

The two most common cases :

- Time series with a d-degree polynomial : ∇^d.
- Time series with a s-period seasonality : ∇_s^D.

Box-Cox transformation

In order to remove a "variance problem", the Box-Cox transformation is sometimes used :

$$\frac{X_t^\lambda - 1}{\lambda}$$

with $\lambda \in \mathbb{R}$.
Note that :

$$\frac{X_t^\lambda - 1}{\lambda} \xrightarrow{\lambda \to 0} \ln\left(X_t\right).$$

Remark

One can find some stationarity tests that aren't comprehensive.

In practice

- A very slowly decrease of the time series “ACF” suggests differencing at lag 1.
- A very slowly decrease of the time series “ACF” every s multiple lags suggests differencing at lag s.

Differencing is iteratively done, and generally : $d \leq 2$ and $D \leq 2$.

Table of contents

Methods

We consider now that the transformed (or not) time series can potentially be fitted by a zero-mean ARMA model.
To select the model order, one can use :

- confidence intervals,
- corner method,
- information criterium,
- heuristic.

Use of confidence intervals

Idea : empirically establish maximum values for p and q.

- For a $\mathrm{AR}(p)$ process :

$$\forall h > p : \sqrt{n}\widehat{r}(h) \xrightarrow{\mathcal{L}} \mathcal{N}(0,1).$$

One can define a confidence interval of level 95% and search the number of lags for which 95% of the $\widehat{r}(h)$ are in $\left[-\frac{1.96}{\sqrt{n}}, \frac{1.96}{\sqrt{n}}\right]$.

- For a $\mathrm{MA}(q)$ process :

$$\forall h > q : \sqrt{n}\widehat{\rho}(h) \xrightarrow{\mathcal{L}} \mathcal{N}\left(0, 1 + 2\sum_{k=1}^{q} \rho^2(k)\right).$$

One can define a confidence interval of level 95% and search the number of lags for which 95% of the $\widehat{\rho}(h)$ are in :

$$\left[-\frac{1.96}{\sqrt{n}}\left(1 + 2\sum_{k=1}^{q} \widehat{\rho}^2(k)\right)^{\frac{1}{2}}, \frac{1.96}{\sqrt{n}}\left(1 + 2\sum_{k=1}^{q} \widehat{\rho}^2(k)\right)^{\frac{1}{2}}\right].$$

Information criterium

Aim : search a model for which a chosen information criterium is minimal.

Heuristic

In order to draw hypothesis, find “significant” autocorrelations :

- For the AR part : partial autocorrelations.
- For the MA part : (simple) autocorrelations.

Table of contents

Estimation

Use of the maximum likelihood estimation after a preliminary estimation.

Table of contents

Aim

- significance test of the parameters,
- whiteness and normality of the residuals.

Significance test of the parameters

For example, we consider the following test for φ_p :

$$\begin{cases} H_0 : \text{le processus est un ARMA}\,(p-1,q)\ (\varphi_p = 0) \\ H_1 : \text{le processus est un ARMA}\,(p,q)\ (\varphi_p \neq 0) \end{cases}.$$

We use the student statistic :

$$t = \frac{|\widehat{\varphi}_p|}{\sqrt{\widehat{\text{Var}}\,(\widehat{\varphi}_p)}}.$$

We reject H_0 at the 5% level if $|t| > 1.96$.

Whiteness and normality of the residuals

- Whiteness : Ljung–Box test.
- Normality : Shapiro-Wilk.

Note that if the normality of the residuals is rejected, it could be useful to add an ARCH or GARCH part. . .

Table of contents

Final order selection

There are two main ways to finally select a model :

- Information criterium (e.g. AIC or BIC)).
- Predictive power.

Table of contents

Forecasting

The forecasting and the forecasting interval are obtained using the autoregressive and moving average representations.

Table of contents

Ex post analysis

Idea : truncate the time series and measure the forecasting error with an indictor such as the Root Mean Square Error (RMSE) or the Mean Average Percentage Error (MAPE) :

$$\text{RMSE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(x_t - \widehat{x}_t)^2},$$

$$\text{MAPE} = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{x_t - \widehat{x}_t}{x_t}\right|.$$