A Generalization of Borwein-Preiss Variational Principle for Set-Valued Mappings

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Abstract. Concerning the conventional line in developing variational principles, observe that the minimization condition in Ekeland’s variational principle 1974 can be interpreted as follows: for every l.s.c. function $f : X \to \mathbb{R} \cup \{+\infty\}$ with $\inf f > -\infty$ there exists a function $\tilde{f} : X \to \mathbb{R} \cup \{+\infty\}$ that supports $f$ from below at some point $\bar{x} \in \text{dom} f$, i.e.,

$$\tilde{f}(\bar{x}) = f(\bar{x}) \quad \text{and} \quad \tilde{f}(\bar{x}) \leq f(x), \forall x \in X.$$ 

Then Ekeland’s principle ensures, in the framework of arbitrary Banach spaces, that the support $\tilde{f}(.)$ can be chosen as a small perturbation by functions of the norm type. A clear disadvantage of this results is the intrinsic nonsmoothness of such perturbations, and so a natural question arises about conditions ensuring smooth perturbations, i.e., about smooth variational principles.

We give a generalization of Borwein-Preiss variational principle for set-valued mappings, replacing the distance and the norm by a "gauge-type" lower semi-continuous function. For set-valued mappings we consider a kind of minimizers different from the Pareto one.