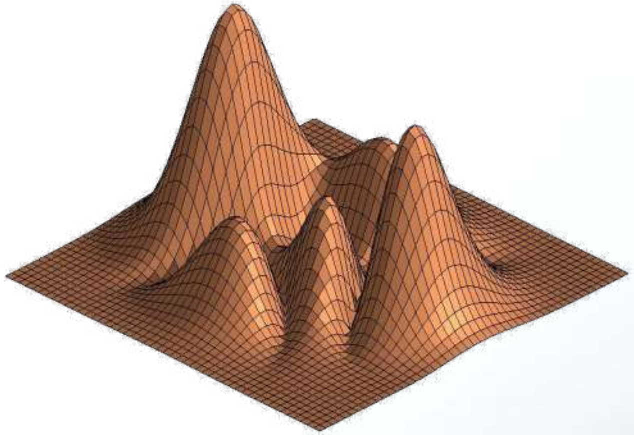
Inference with Multi-Object Hidden Markov Models



Ba-Ngu Vo http://ba-ngu.vo-au.com
Dept. Electrical & Computer Engineering
Curtin University
Perth, Australia

ISPL
Intelligent Sensing & Perception Lab
https://ispl-research.com/

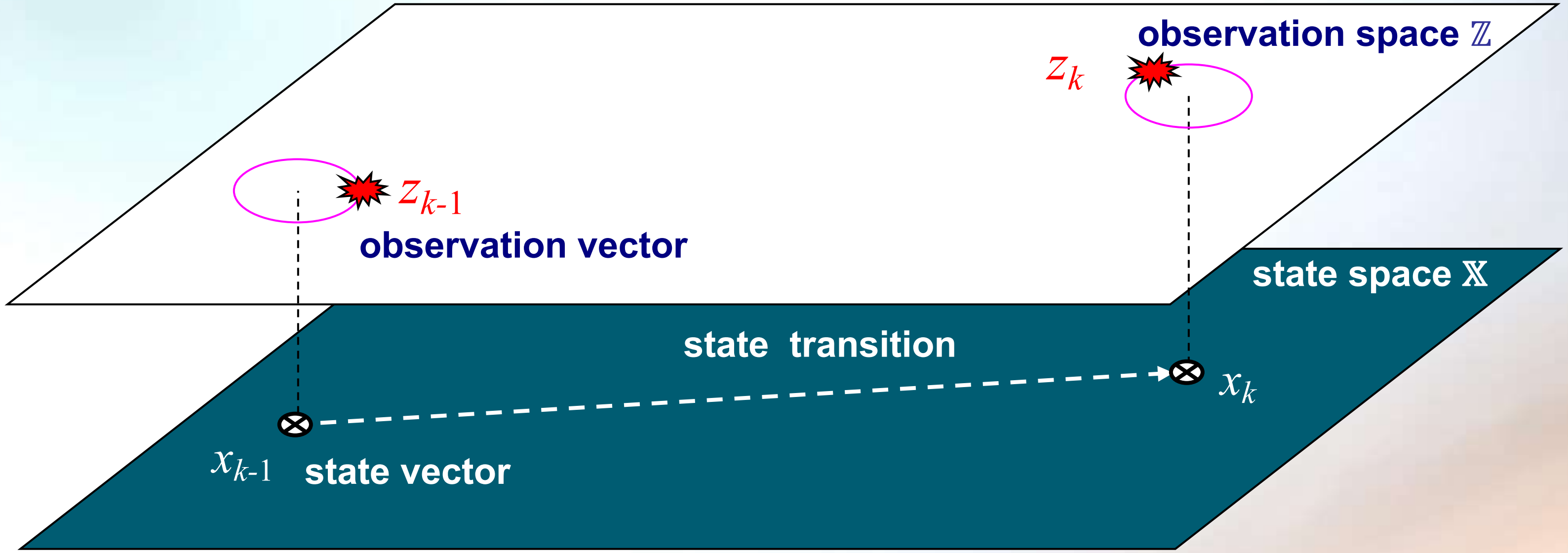
VIASM November 2022

Outline

- Multi-Object Hidden Markov Model (HMM)
- Stochastic Geometry
- Inference for Multi-Object HMM
- Generalized Labeled Multi-Bernoulli
- Applications
- Conclusions

Multi-Object Hidden Markov Model (HMM)

Hidden Markov Model (HMM) aka State Space Model of a Dynamical System



- State Estimation
- System ID
- Control

State equation $x_k = \Phi(x_{k\text{-}1}, u_{k\text{-}1}, n_k)$ → $f_{k|k\text{-}1}(x_k | x_{k\text{-}1}, u_{k\text{-}1})$

Markov transition density

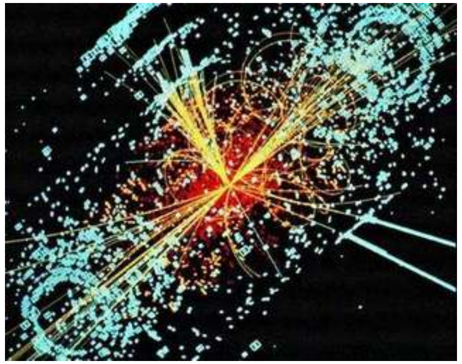
control noise

Observation likelihood

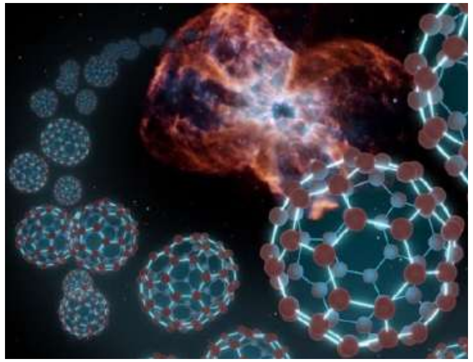
Observation equation $z_k = \Psi(x_k, u_k, v_k)$ → $g_k(z_k | x_k, u_{k\text{-}1})$

Multi-Object Hidden Markov Model (HMM)

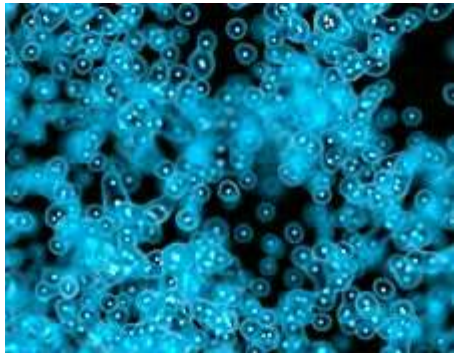
- What if the state & observation are not vectors but point patterns, i.e., finite sets?



Particles



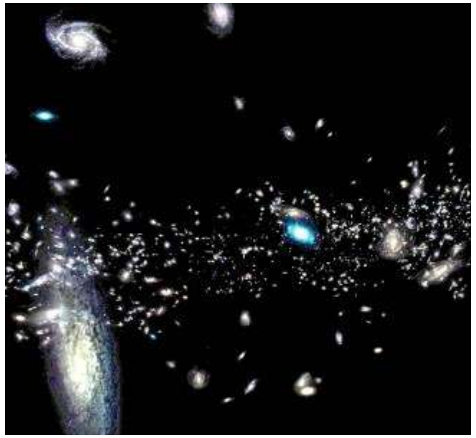
Molecules



Cells

People

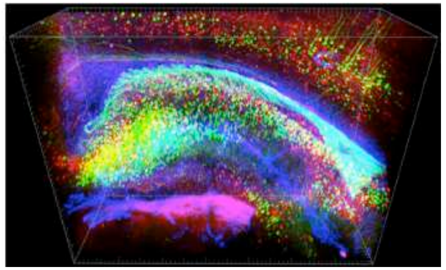
Orbital Debris



Astronomy

Multi-Object Hidden Markov Model (HMM)

- **Point patterns in data science**



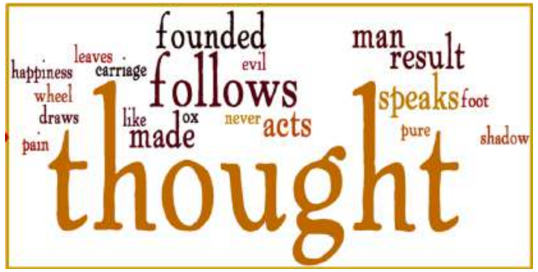
Neuron point cloud

Tree locations

5	0	0	0	0	0	0	0	{(1,5)}
0	0	0	0	0	0	0	37	{(8,37)}
6	5	0	0	0	0	0	6	{(1,6), (2,5), (8,6)}
80	0	0	0	0	0	0	0	{(1,80)}
0	0	0	6	0	0	0	0	{(4,6)}
48	0	0	0	0	0	0	26	{(1,48), (8,26)}

Sparse data

SIFT key points



Bag of words

Transaction record

- Multiple Instance (MI) Learning: Machine Learning for point patterns [Amores 13]

Stochastic Geometry

Bayesian: $p(X|\,\text{Data}\,)$ Posterior PDF

including finite sets

Non-Bayesian: $p(\,\text{Data}\,|X\,)$ Data likelihood function

- **What's the big deal about working with sets?**

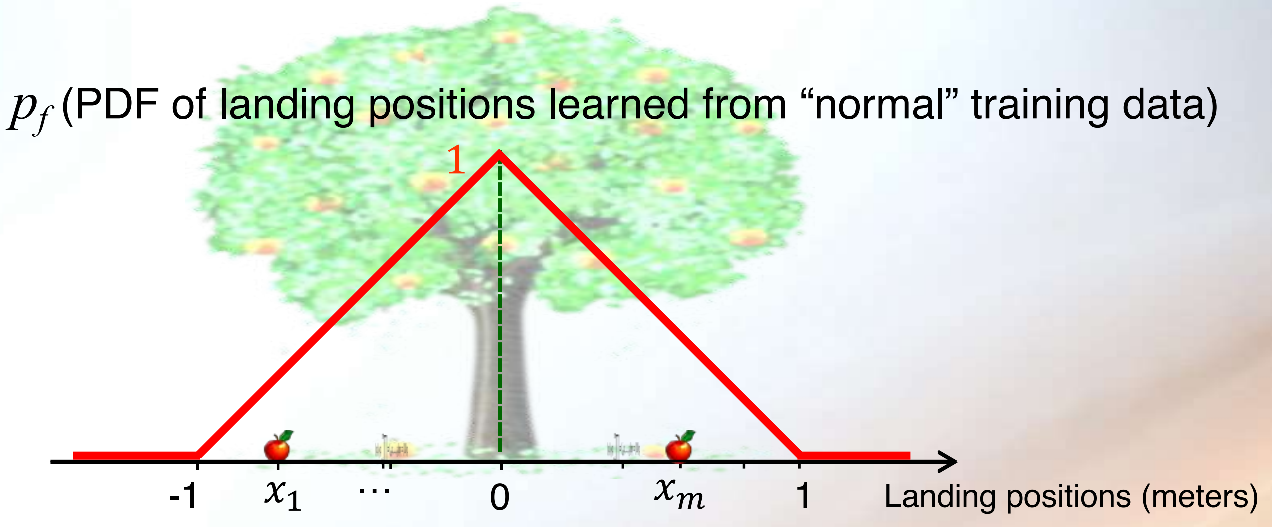
  Most widely adopted practice (in engineering/computer science):

$X = \{x_1, \ldots, x_m\}$ ⇒ $p(X) = p(x_1, \ldots, x_m)$

Finite Set — Probability Density Function (PDF)

Stochastic Geometry

What's the big deal about working with sets?



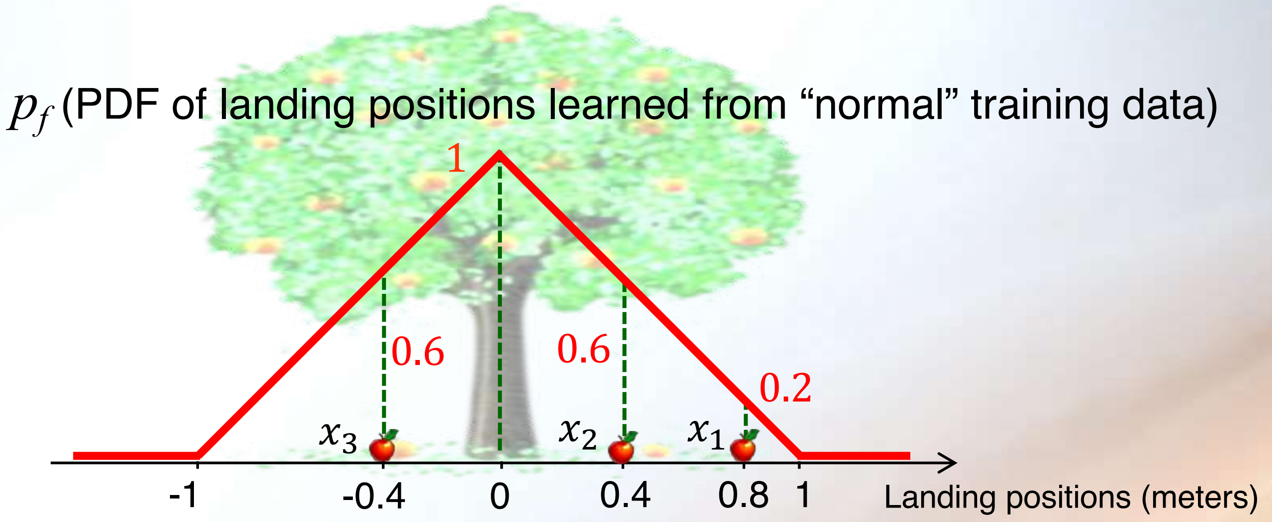
- Apples land on the ground independently from each other

$$p(X) = p(x_1, \dots, x_m) = \prod_{i=1}^{m} p_f(x_i)$$

- Daily landing patterns are independent from each other
- **Novelty Detection:** find unlikely daily landing patterns

Stochastic Geometry

What's the big deal about working with sets?



Day 1: x_1 $$p(x_1) = p_f(x_1) = 0.2,$$

Day 2: x_2, x_3 $$p(x_2, x_3) = p_f(x_2)\, p_f(x_3) = 0.36$$

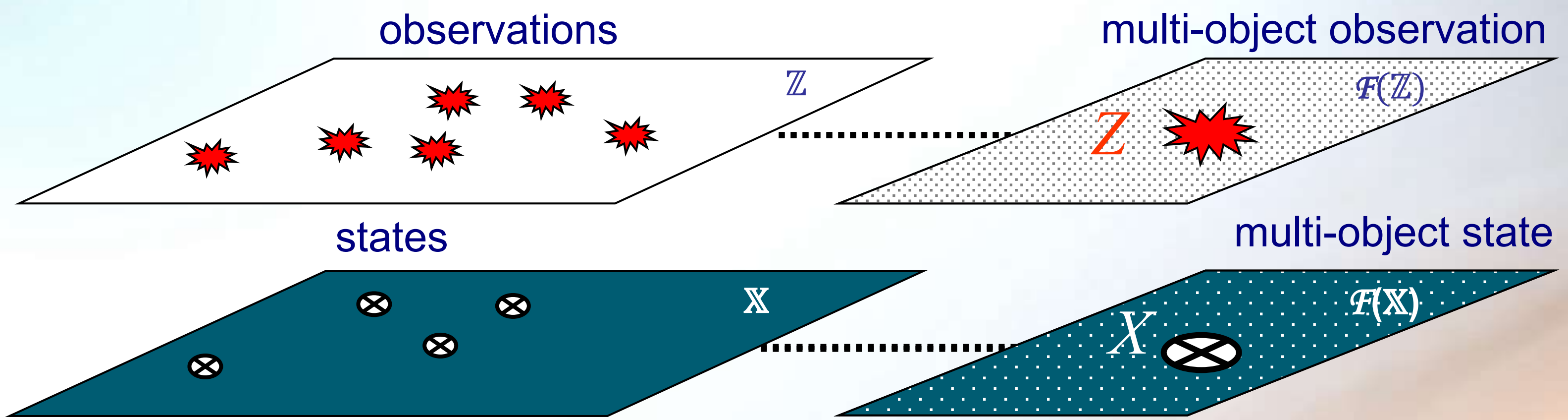
- Q: Which pattern is less likely? Ans: day 1 less likely than day 2.
- Change unit of measurement from m to cm and …

$$p(x_1) = 0.002 > p(x_2, x_3) = 0.000036$$

BN. Vo et. al. "Model-based learning for point pattern data" Pattern Recognition 84, 136-151, 2018

Stochastic Geometry

- **Multi-Object HMM**: HMM where the state is a finite set - multi-object state



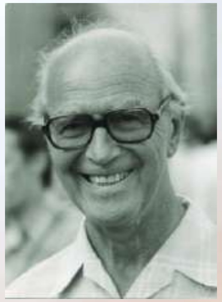
- **Model multi-object state as a random finite set (RFS) …**
  - Needs: Markov transition density & observation likelihood for finite-set-valued **state**
  - Can’t treat a (random) finite set as if it were a (random) vector
  - PDF of random finite set not the same as PDF of random vector
  - **Need PDFs (+** suitable notion of density & integration**) for Random Finite Sets**

Stochastic Geometry

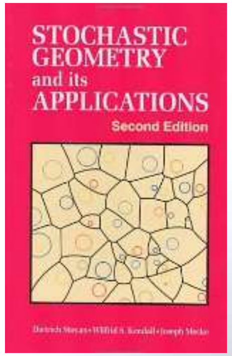
- Mathematical tools for dealing with Random Sets
- Foundation (1960s-1970s): mostly due to independent work by **Matheron** and **Kendall**, both of whom gave credits to earlier work by **Choquet**

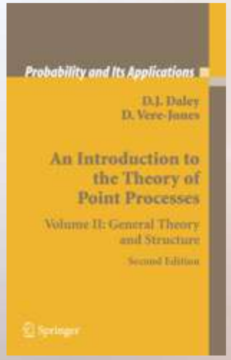
G. Matheron (1930-2000)

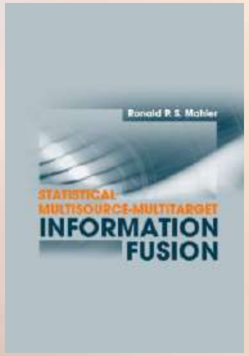
D. Kendall (1918-2007)



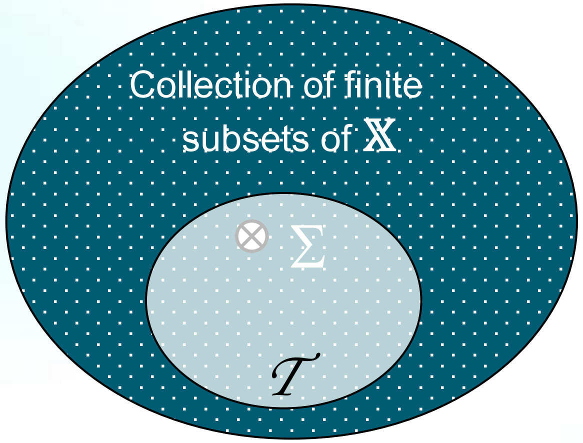
G. Choquet (1915-2006)

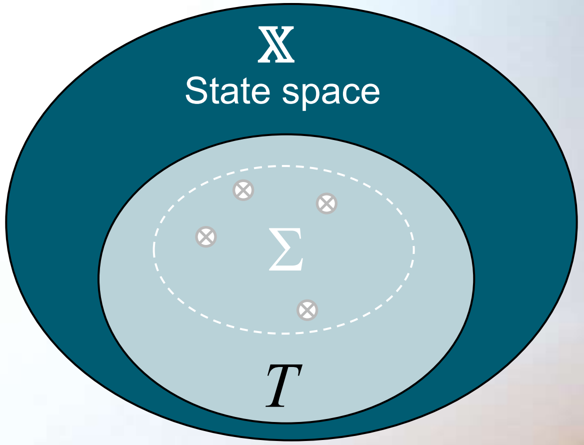






Stochastic Geometry





Probability distribution of Σ

$$P_\Sigma(\mathcal{T}) = P(\Sigma \in \mathcal{T}),\ \mathcal{T} \subseteq \mathcal{F}(\mathbb{X})$$

Point Process Theory (1950-1960's)

Choquet (1968)

Belief "distribution" of Σ

$$\beta_\Sigma(T) = P(\Sigma \subseteq T),\ T \subseteq \mathbb{X}$$

Mahler's Finite Set Statistics (1994)

Probability density of Σ

$$p_\Sigma : \mathcal{F}(\mathbb{X}) \to [0,\infty)$$

$$P_\Sigma(\mathcal{T}) = \int_{\mathcal{T}} p_\Sigma(X)\mu(dX)$$

Conventional integral

VSD (2005)

Belief "density" of Σ

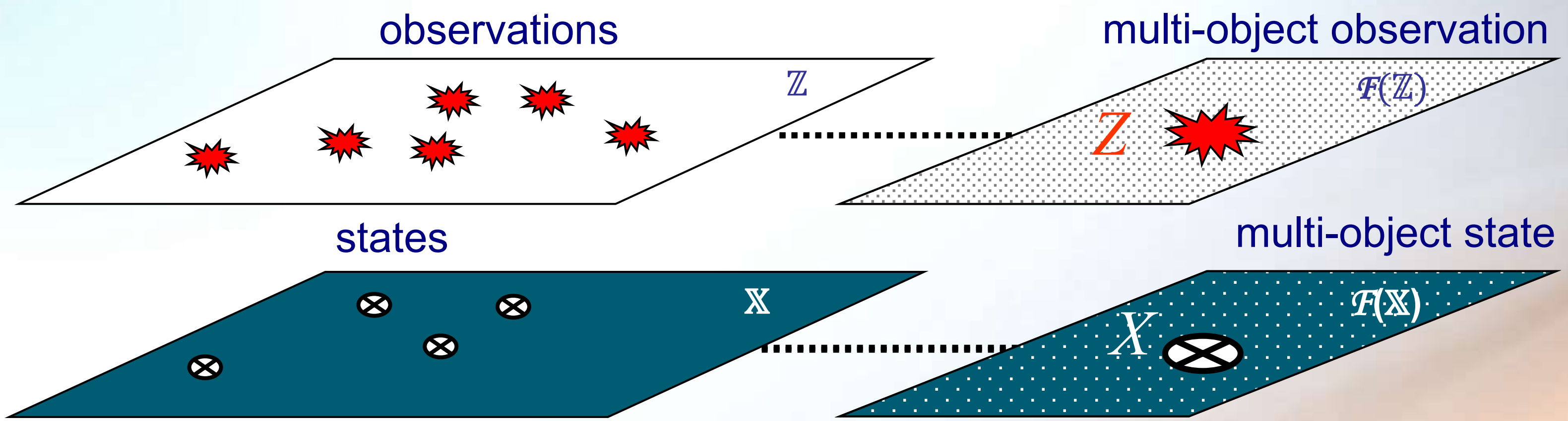
$$f_\Sigma : \mathcal{F}(\mathbb{X}) \to [0,\infty)$$

$$\beta_\Sigma(T) = \int_T f_\Sigma(X)\delta X$$

Set integral

Inference for Multi-Object HMM

- **Multi-Object HMM**: Finite-set-valued HMM

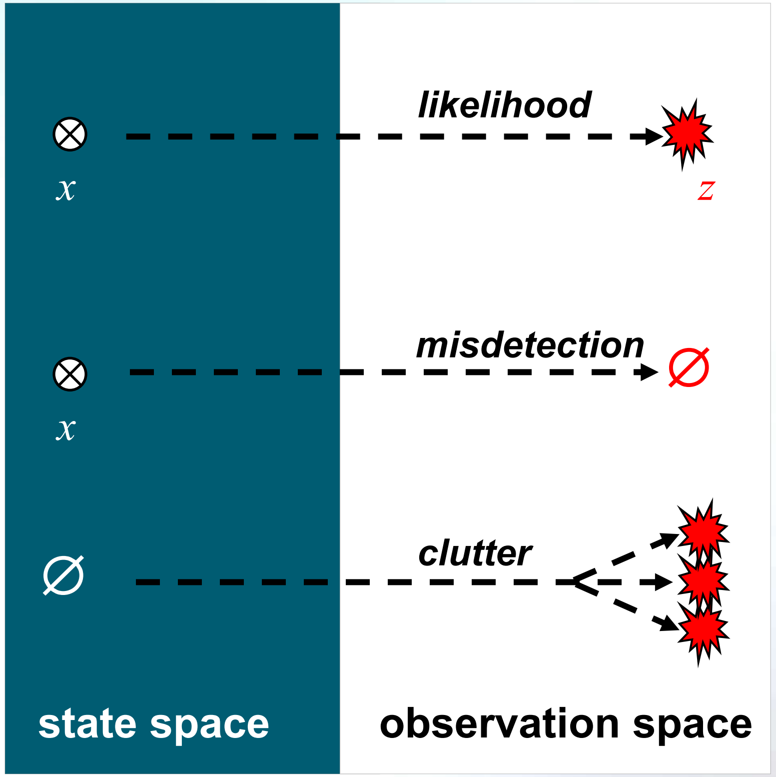


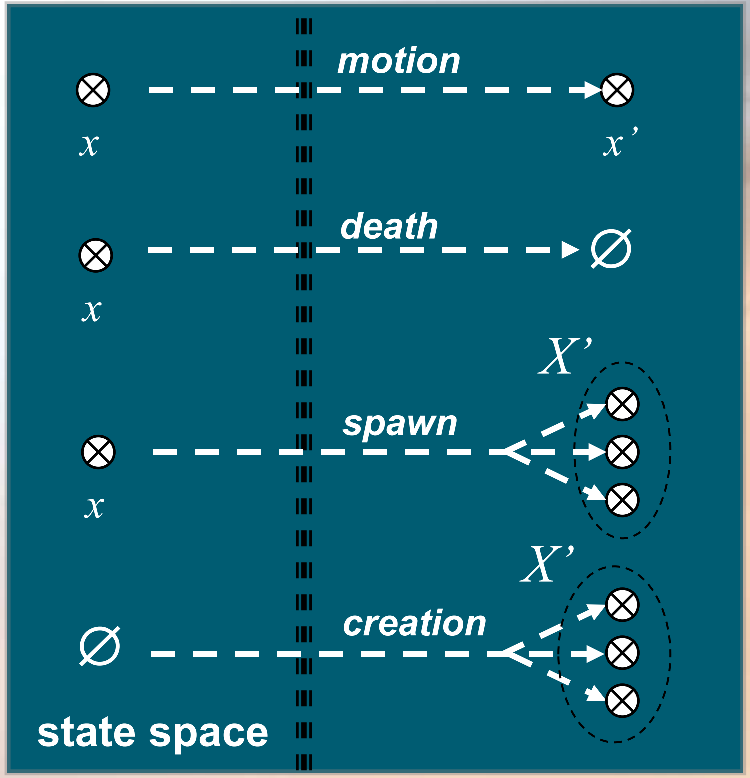
- **Inference for Multi-Object HMM**: State Estimation on the space $\mathcal{F}(\mathbb{X})$ of finite sets of $\mathbb{X}$
- **Fundamental difference from classical dynamical system:**
  - Random time-varying number of states and measurements
  - False negatives, false positives, association uncertainty
  - **Much more challenging computationally!**

R. Mahler, *Statistical Multisource-Multitarget Information Fusion*, Artech House, 2007.
R. Mahler, *Advances in Statistical Multisource-Multitarget Information Fusion*, Artech House, 2014.

Inference for Multi-Object HMM

A Standard Multi-Object HMM

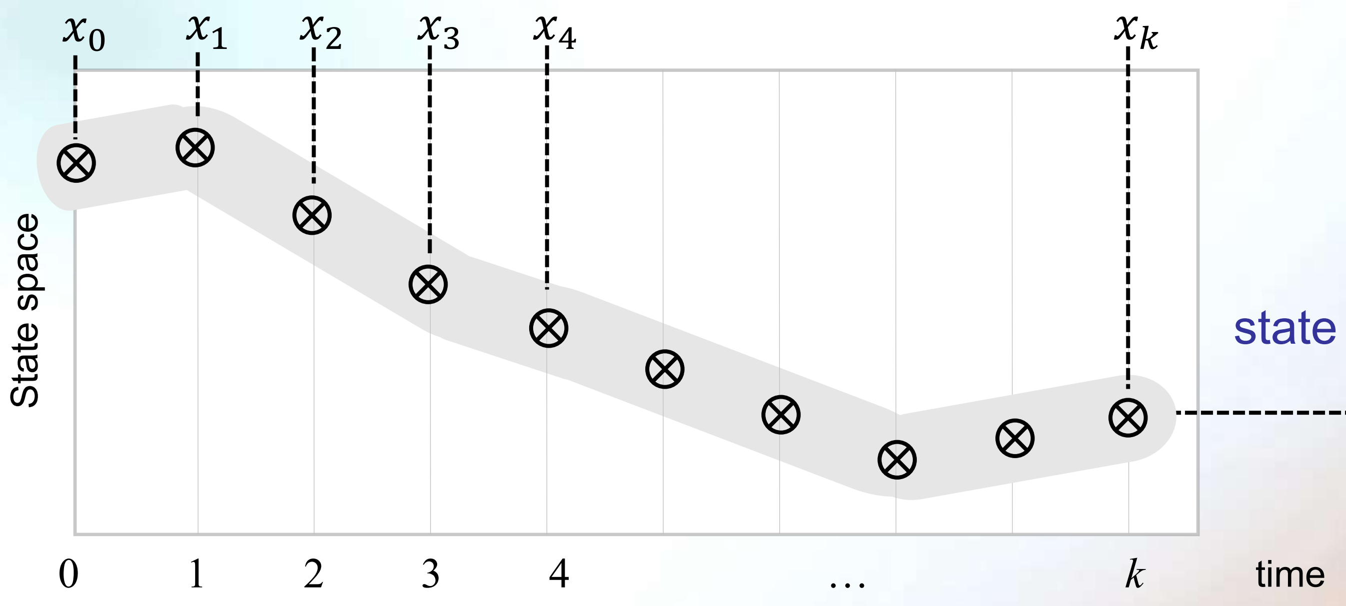




Observation likelihood $g_k(Z_k|X_k)$

State transition kernel $f_{k|k-1}(X_k|X_{k-1})$

Inference for Multi-Object HMM



state trajectory = history of states

$x_{0:k} = [x_0, \ldots, x_k]$

- **State Estimation**: estimate state trajectory
- **Online Operation**: need **fixed complexity per time step** to be useful in practice
- **Filtering**: $\widehat{x_0}, \ldots, \widehat{x_k}$, **suitable** for **online** – Kalman & particle filters
- **Smoothing**: $\widehat{x_{0:k}}$, **not suitable** for **online** – Kalman & particle smoothers

S. Sarkka, Bayesian filtering and smoothing, Cambridge University Press, 2013

Inference for Multi-Object HMM

$$\frac{\boxed{g_k(z_k|\,x_k) f_{k|k-1}(x_k|\,x_{k-1})}\; p_{0:k-1}(x_{0:k-1}\,|z_{1:k-1})}{\int g_k(z_k|\,x_k)\, f_{k|k-1}(x_k|\,x_{k-1}) p_{0:k-1}(x_{0:k-1}|z_{1:k-1}) dx_{0:k}}$$

smoothing-while-filtering

Bayes smoother

$$\cdots \longrightarrow p_{0:k-1}(x_{0:k-1}\,|z_{1:k-1}) \longrightarrow p_{0:k}(x_{0:k}|\,z_{1:k}) \longrightarrow \cdots$$

- Complexity per time step increases with time - **not suitable** for **online**
- Smooth over a fix-length moving window - fixed complexity per time step

$$\int p_{k-1}(x_{k-1}|\,z_{1:k-1}) \boxed{f_{k|k-1}(x_k|\,x_{k-1})}\, dx_{k-1}$$

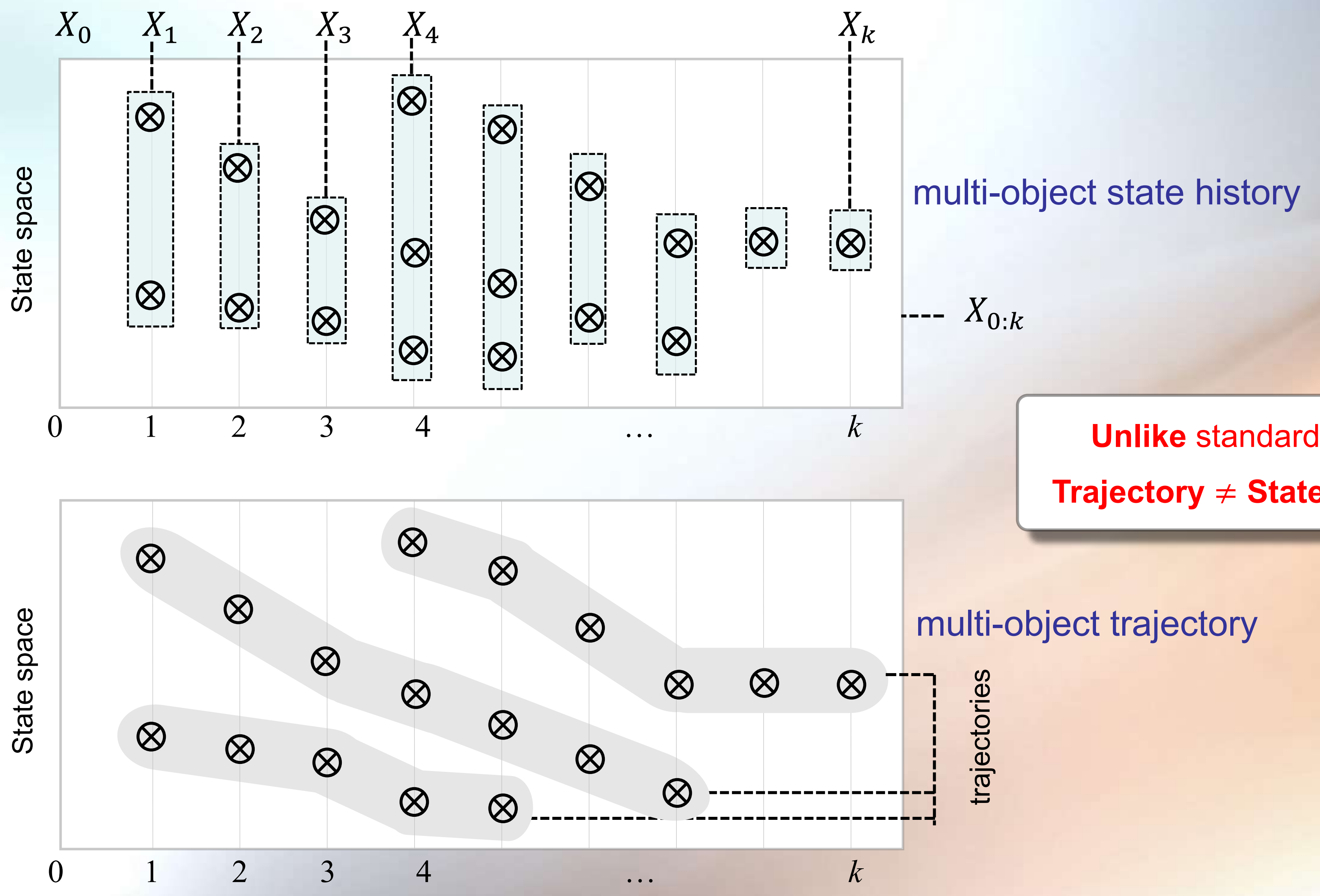
$$\frac{\boxed{g_k(z_k|\,x_k)}\; p_{k|k-1}(x_k|\,z_{1:k-1})}{\int g_k(z_k|\,x_k) p_{k-1}(x_{k-1}|\,z_{1:k-1}) dx_k}$$

Bayes filter

$$\cdots \longrightarrow p_{k-1}(x_{k-1}\,|z_{1:k-1}) \xrightarrow{\text{prediction}} p_{k|k-1}(x_k|\,z_{1:k-1}) \xrightarrow{\text{data-update}} p_k(x_k|\,z_{1:k}) \longrightarrow \cdots$$

- Fixed complexity per time step - **suitable** for **online**, widely used

Inference for Multi-Object HMM

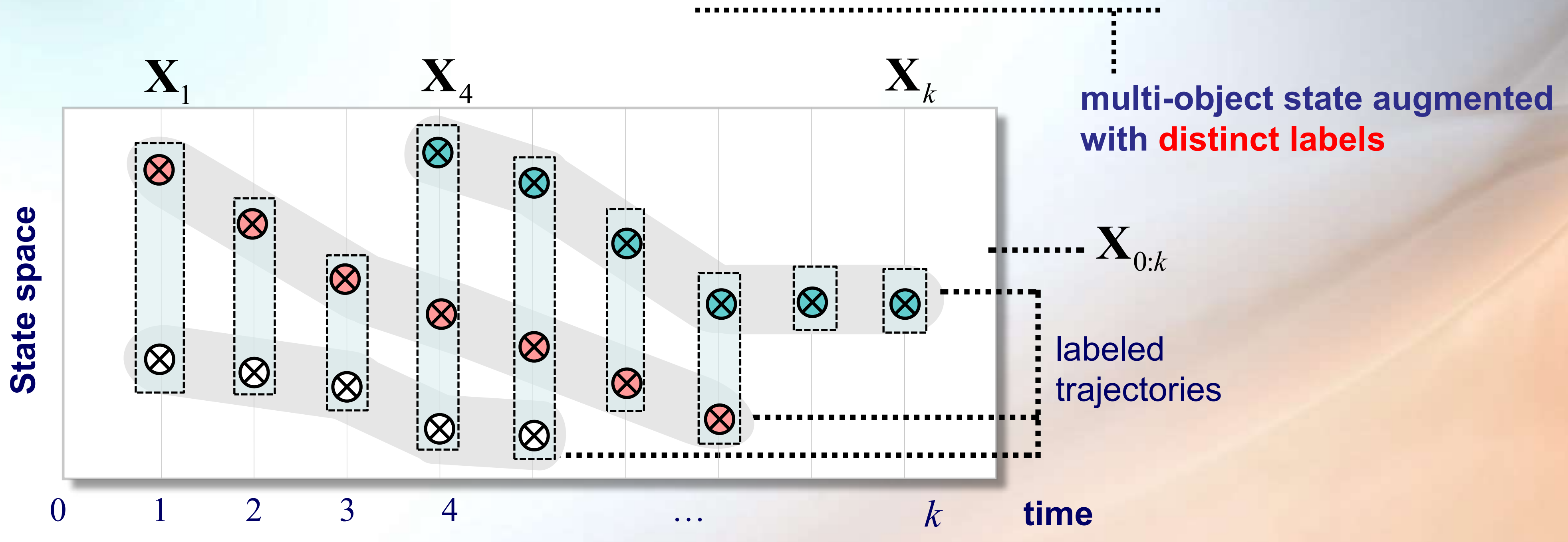


Unlike standard HMM

Trajectory $\neq$ State History

Inference for Multi-Object HMM

- **multi-object trajectory = history of labeled multi-object states**



- Labels provides multi-object trajectory estimate (even from a single scan)
  - filtering with labeled multi-object states: $\hat{\mathbf{X}}_0, \dots, \hat{\mathbf{X}}_k$
  - smoothing with labeled multi-object states: $\widehat{\mathbf{X}_{0:k}}$

Inference for Multi-Object HMM

- Labels provides multi-object trajectory estimate (even from a single scan)
  - filtering with labeled multi-object states: $\hat{\mathbf{X}}_0,\ldots,\hat{\mathbf{X}}_k$
  - smoothing with labeled multi-object states: $\widehat{\mathbf{X}_{0:k}}$

- Labels admits closure under posterior truncation

multi-object likelihood

$$g_k(Z|\mathbf{X}) \propto \sum_{\theta} \left[\Psi_{Z,k}^{(\theta(\mathcal{L}(\cdot))}(\cdot)\right]^{\mathbf{X}}$$

Vo & Vo "Labeled RFSs and Multi-Object Conjugate Priors," IEEE Trans. SP, 61(13): 3460-3475, 2013

  - **Each term** of the multi-object likelihood is **symmetric**
  - **Truncated** labeled posterior/filtering density is a **function of sets**

- **2 birds with one stone:** provides **trajectories & closure under truncation**

Generalized Labeled Multi-Bernoulli (GLMB)

- **GLMB filter**: Multi-object Analogue of Kalman Smoother/Filter
- **Closure under Bayes recursion -** Analytic solutions to:
  - Multi-object Bayes Posterior Recursion (Smoothing-while-filtering)

$$\boldsymbol{\pi}_{0:k}(\mathbf{X}_{0:k}|Z_{1:k}) \propto g_k(Z_k|\mathbf{X}_k)\mathbf{f}_{k|k-1}(\mathbf{X}_k|\mathbf{X}_{k-1})\boldsymbol{\pi}_{0:k-1}(\mathbf{X}_{0:k-1}|Z_{1:k-1})$$

estimated **labeled** multi-object state history $\widehat{\mathbf{X}_{0:k}}$

Vo & Vo "A Multi-Scan Labeled RFS Model for Multi-object State Estimation," IEEE Trans. SP, 67(19):4948-4963, 2019

  - Multi-object Bayes Filtering Recursion

$$\boldsymbol{\pi}_{k|k-1}(\mathbf{X}_k) = \int \mathbf{f}_{k|k-1}(\mathbf{X}_k|\mathbf{X})\boldsymbol{\pi}_{k-1}(\mathbf{X})\delta\mathbf{X},$$

$$\boldsymbol{\pi}_k(\mathbf{X}_k) = \frac{g_k(Z_k|\mathbf{X}_k)\boldsymbol{\pi}_{k|k-1}(\mathbf{X}_k)}{\int g_k(Z_k|\mathbf{X})\boldsymbol{\pi}_{k|k-1}(\mathbf{X})\delta\mathbf{X}},$$

estimated **labeled** multi-object states $\widehat{\mathbf{X}}_0,\ldots,\widehat{\mathbf{X}}_k$ forms a set of tracks

Vo & Vo "Labeled RFSs and Multi-Object Conjugate Priors," IEEE Trans. SP, 61(13): 3460-3475, 2013

Generalized Labeled Multi-Bernoulli (GLMB)

- GLMB density - multi-object analogue of exponential mixture:

$$\boldsymbol{\pi}(\mathbf{X}) = \underbrace{\Delta(\mathbf{X})}_{\delta_{|\mathbf{X}|}(|\mathcal{L}(\mathbf{X})|)} \sum_{\xi \in \Xi} w^{(\xi)}(\underbrace{\mathcal{L}(\mathbf{X})}_{\text{labels of } \mathbf{X}}) \underbrace{\left[p^{(\xi)}\right]^{\mathbf{X}}}_{\prod_{\mathbf{x}\in\mathbf{X}} p^{(\xi)}(\mathbf{x})}$$

distinct label indicator associations history multi-object exponential

- **Weight Normalization**

$$\sum_{L \subseteq \mathbb{L}} \sum_{\xi \in \Xi} w^{(\xi)}(L) = 1 \qquad \int p^{(\xi)}(x, \ell) dx = 1$$

- **Cardinality Distribution & 1st moment**

$$\rho(n) = \sum_{\xi \in \Xi} \sum_{L \subseteq \mathbb{L}} \delta_n(|L|) w^{(\xi)}(L) \qquad v(x,\ell) = \sum_{\xi \in \Xi} p^{(\xi)}(x,\ell) \sum_{L \subseteq \mathbb{L}} 1_L(\ell) w^{(\xi)}(L)$$

B.-T. Vo, et. al. "Labeled Random Finite Sets and Multi-Object Conjugate Priors," IEEE Trans. SP, 61(13): 3460-3475, 2013.

Generalized Labeled Multi-Bernoulli (GLMB)

Labeled Multi-Object Density Approximations

Truncation of GLMB

- Any truncation of a GLMB is a GLMB – closure under truncation
- L1-norm of truncation error can be computed analytically
- Minimum L1-norm truncation error: truncate smallest weights

Vo et. al. "Labeled RFS and the Bayes Multi-Target Tracking Filter," IEEE Trans. SP, 62(24):6554-6567, 2014

Approximation of GLMB by a 1-term GLMB (or LMB) with

- Same PHD & same Cardinality distribution

Reuter et. al. "The labelled multi-Bernoulli filter," IEEE Trans. SP, 62(12):3246-3260, 2014

Approximation of labeled multi-object density by a GLMB with:

- Same PHD & same Cardinality distribution
- Minimal Kullback-Leibler divergence

Papi et. al., “GLMB approximation of Multi-object densities,” IEEE Trans. SP, 63(20):5487-5497, 2015

Generalized Labeled Multi-Bernoulli (GLMB)

- GLMB Filtering/Posterior sum grows in no. terms, requires reduction of terms
- Truncate terms with smallest weights ⇒ minimum L1-norm truncation error
- **Implementation**: How to truncate without exhaustive enumeration of the terms?

- **Filter implementation**:
  - K-shortest path prediction & ranked assignment update (cubic in no. detections)
  - Gibbs sampling + Joint Prediction & Update (linear in no. detections)

Vo et. al. "Labeled RFS and the Bayes Multi-Target Tracking Filter," IEEE Trans. SP, 62(24):6554-6567, 2014.
Vo et. al. "An Efficient Implementation of the GLMB Filter," IEEE Trans. SP, 65(8):1975-1987, 2017.

- **Multi-sensor filter implementation**:
  - Multi-dimensional assignment (NP-Hard): Gibbs sampling (linear in total no. of detections)

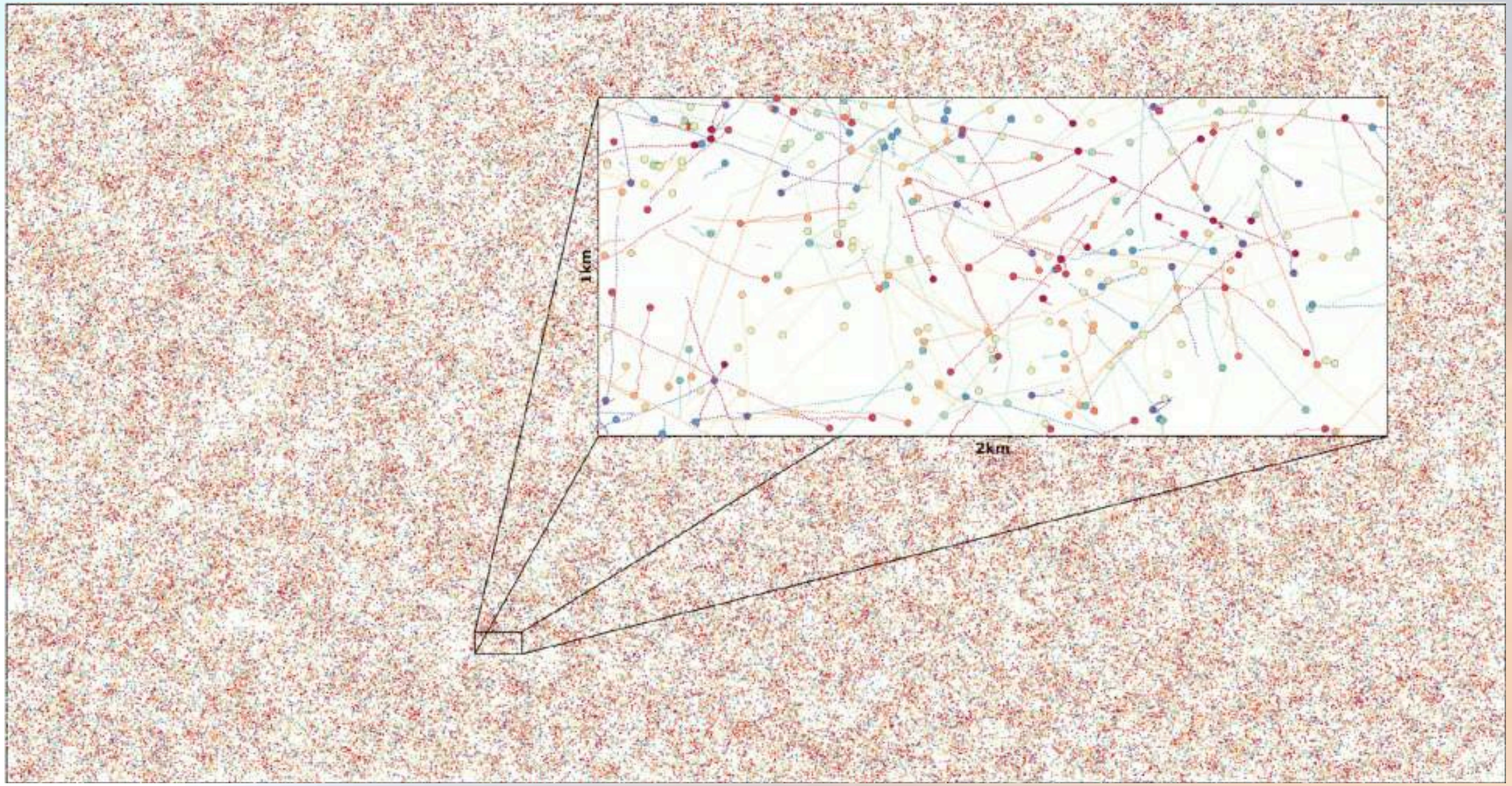
Vo et. al. "Multi-sensor multi-object tracking with the GLMB filter," IEEE Trans. SP, 67 (23):5952-5967, 2019.

- **Smoother implementation**:
  - Multi-dimensional assignment (NP-Hard): Gibbs sampling (linear in total no. of time steps)
  - On-line: smooth over fixed-length window and link trajectories with the same labels

Vo & Vo "A Multi-Scan Labeled RFS Model for Multi-object State Estimation," IEEE Trans. SP, 67(19):4948-4963, 2019

Generalized Labeled Multi-Bernoulli (GLMB)

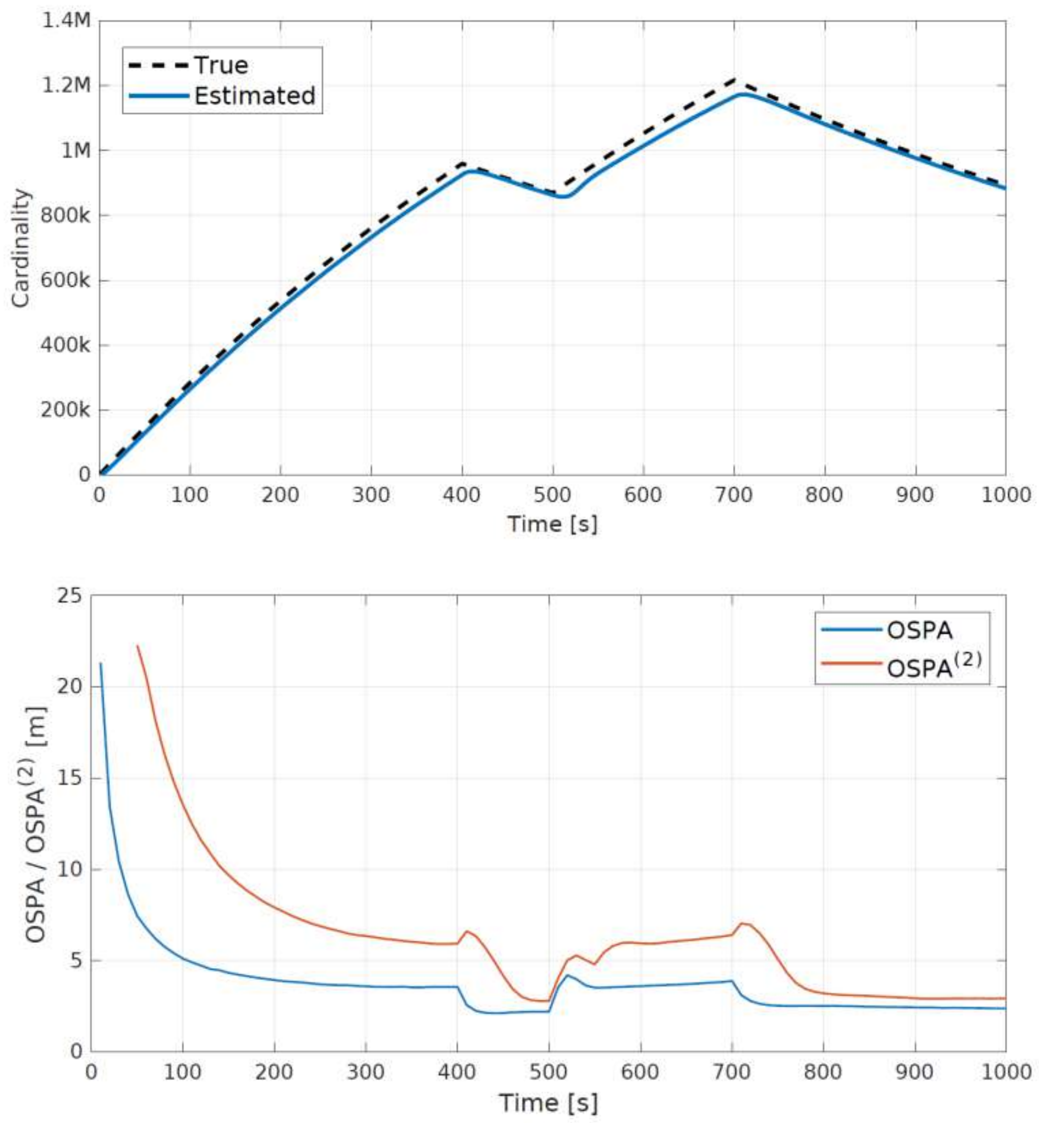
- **Computational Tractability?**



estimated trajectories on a 64km by 32km area

M. Beard et. al. "A Solution for Large-scale Multi-object Tracking," IEEE Trans. SP, 68:2754–2769, 2020.

Generalized Labeled Multi-Bernoulli (GLMB)



Computational Tractability?

Advanced algorithms: at best hundreds of objects/frame

Problem size = no. objects or observations/**frame**

GLMB

- Over **1 million** objects per frame
- Peak cardinality: 1,217,531 objects/frame (at time 700)
- Peak object density: 520 km^{-2}
- Duration 1000 instances … ~ 1 billion data points

- OSPA$^{(2)}$: metric for sets of tracks,
- Generalization of OSPA that accounts for:
  - Localization & Cardinality error;
  - Track fragmentation;
  - ID switches

Beard et. al. "A Solution for Large-scale MOT," IEEE Trans. SP, 68:2754–2769, 2020.

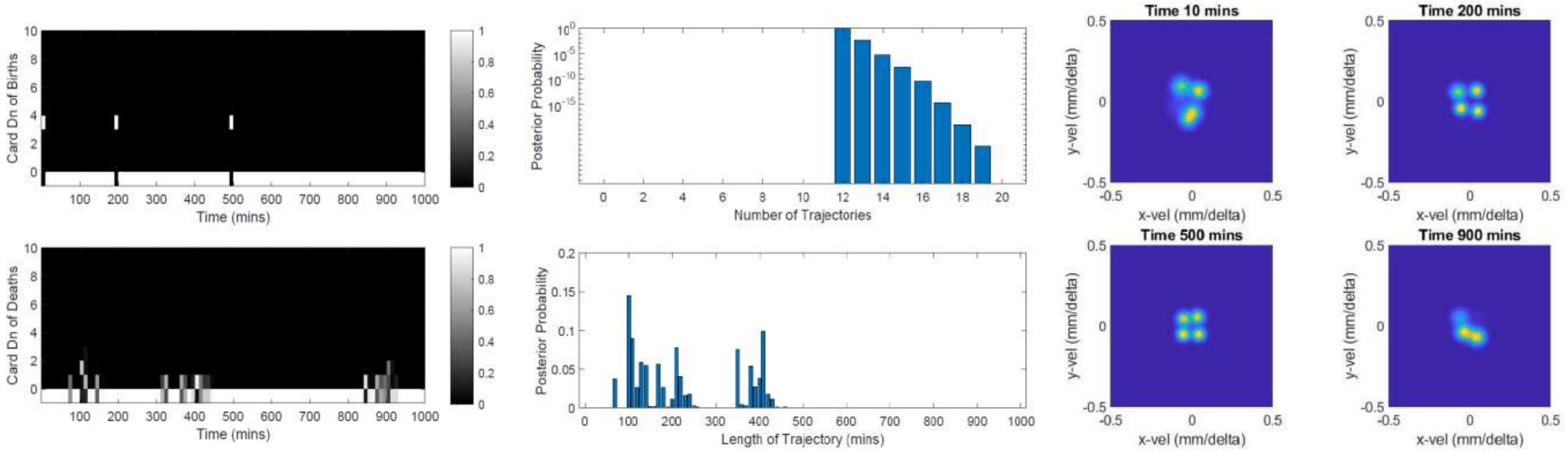
Generalized Labeled Multi-Bernoulli (GLMB)

- Smoother: posterior statistics example in cell-microscopy
  - Duration 1000mins, sampling period Δ=10mins
  - Clutter 0.3/scan,
  - Detection probability 0.33 (very low)
  - Observation noise sigma 0.3mm
  - Dynamic noise sigma 0.01mm/Δ^2

Scenario:

- min 001: 4 cells appear, live for 100mins
- min 200: 4 cells appear, live for 200mins
- min 500: 4 cells appear, live for 400mins

Statistics on births/deaths, cell-life, migration pattern



Vo & Vo “A Multi-Scan Labeled RFS Model for Multi-object State Estimation,” IEEE Trans. SP, 67(19):4948-4963, 2019

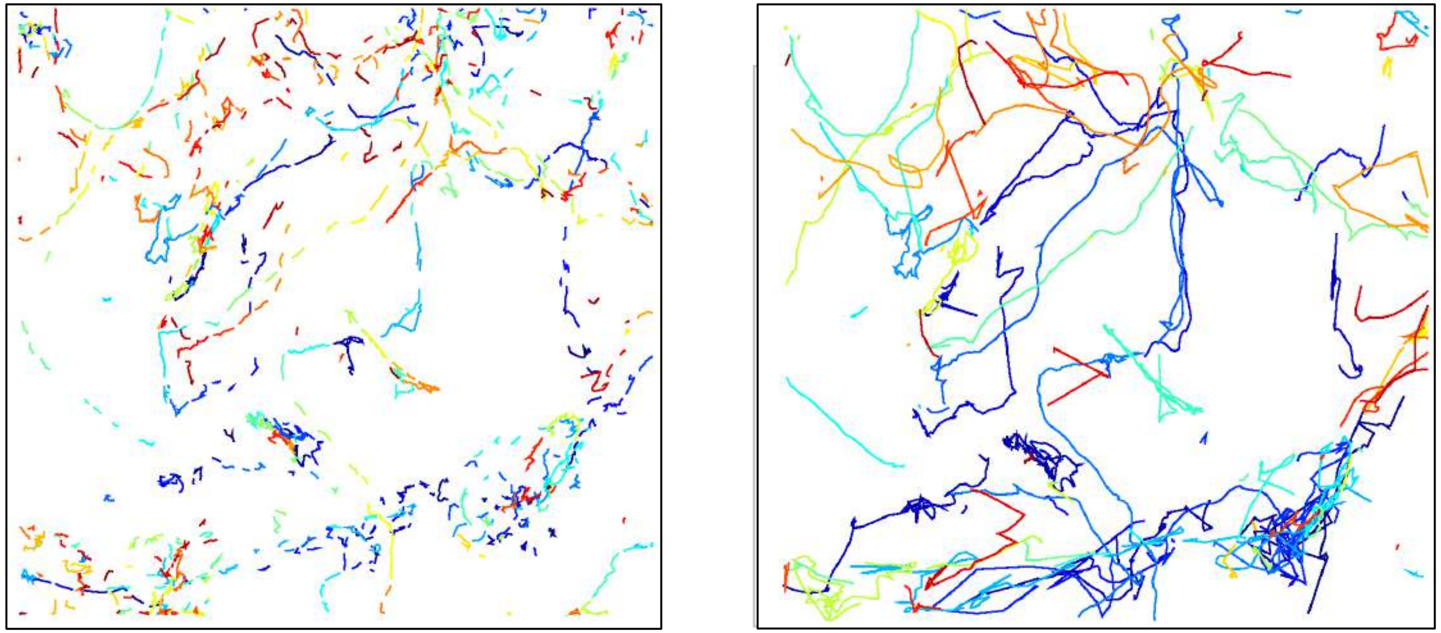
Application: Cell-Microscopy

Cell Migration Analysos

- 4320 mins stem cell migration sequence, 1 image every 16 min

MHT (tree depth=5) [Chenouard et. al.13]

GLMB filter: online & less parameters

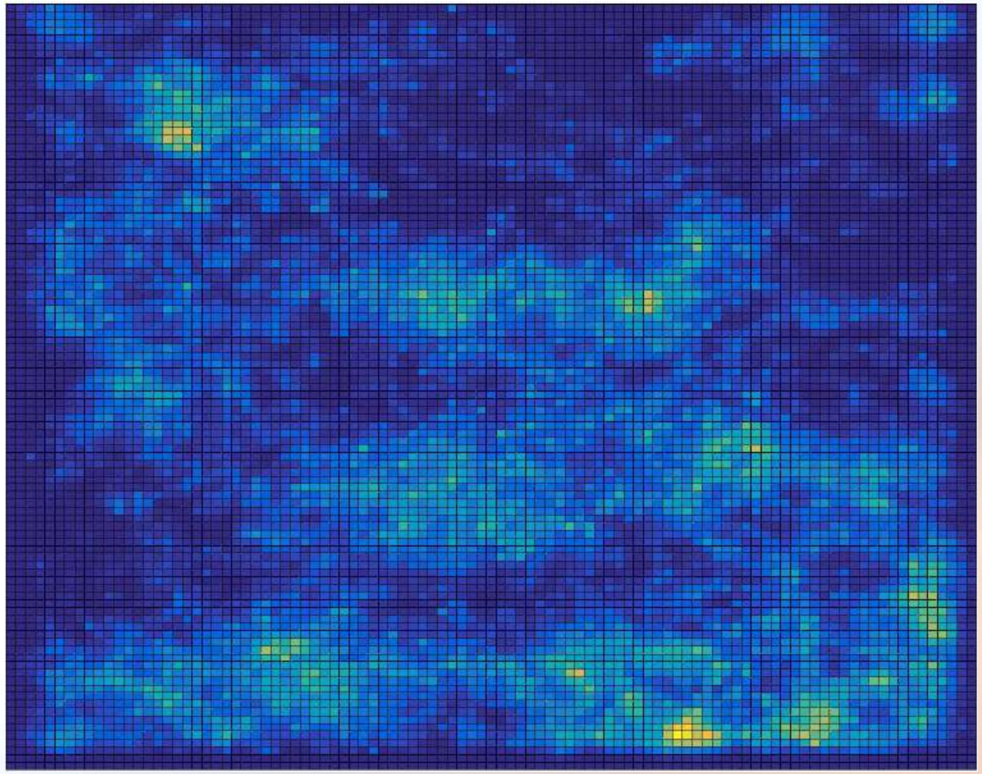


Kim et al. "A GLMB tracker for time lapse cell migration," ICCAIS’17, Thailand, 2017.

Application: Cell-Microscopy

Cell Migration Analysis

- Provides more than just tracking results: Statistics from the posterior distribution of the cells:



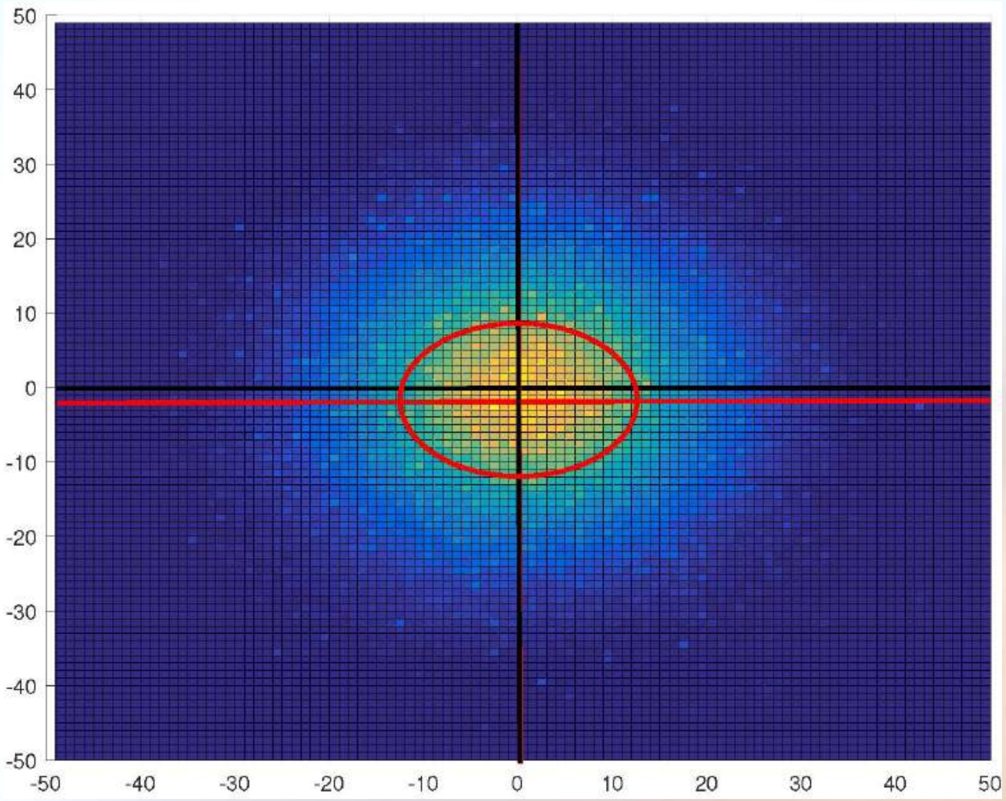
Net migration Top –Bottom!

Time averaged intensity function in position space …. Mean number of cells per unit area

Application: Cell-Microscopy

Cell Migration Analysis

- Provides more than just tracking results: Statistics from the posterior distribution of the cells:



Net migration Top –Bottom!

Time averaged intensity function in velocity space

Interpretation: Velocity heatmap or concentration of cells at different velocities

Application: Multi-Sensor Fusion

- 3D (state + extent) tracking from multiple camera views
  - 4 Kinect sensors placed high up and facing inwards in each corner
  - YOLO detector produces 2D detections of bounding boxes in pixel space
  - Detections are noisy and subject to false positives/negatives

Ong et. al. “A Bayesian Filter for Multi-view 3D Multi-object Tracking with Occlusion Handling,” IEEE Trans. PAMI, 2020.

Application: Multi-Sensor Fusion

- Provides tracks in 3D instead of ground-plane tracks as in existing methods
- No retraining when reconfiguring cameras

CMC3 (Maximum/Average 15 people)													
Detector and Tracker	IDF1 ↑	IDP ↑	IDR ↑	MT ↑	PT ↓	ML ↓	FP ↓	FN ↓	IDs ↓	FM ↓	MOTA ↑	MOTP ↑	OSPA$^{(2)}$ ↓
YOLOv3+MV-GLMB-OC	**70.7%**	**72.3%**	**69.1%**	**14**	**1**	**0**	94	**222**	**45**	**37**	**87.2%**	**52.8%**	**0.53**
YOLOv3+MV-GLMB-OC*	60.8%	65.7%	56.6%	9	6	**0**	91	481	66	56	77.4%	46.4%	0.60
YOLOv3+MS-GLMB	41.4%	57.3%	32.4%	0	15	**0**	**10**	1239	64	60	53.5%	46.7%	0.76
Faster-RCNN(VGG16)+MV-GLMB-OC	63.7%	66.6%	61.1%	12	3	**0**	97	329	63	41	82.7%	**52.8%**	0.58
Faster-RCNN(VGG16)+MV-GLMB-OC*	57.3%	61.0%	54.0%	10	5	**0**	133	460	78	60	76.3%	47.9%	0.66
Faster-RCNN(VGG16)+MS-GLMB	45.7%	61.7%	36.3%	0	15	**0**	13	1175	61	67	55.8%	46.6%	0.75
CMC5 (Jumping and Falling, Maximum/Average 7 people)													
Detector and Tracker	IDF1 ↑	IDP ↑	IDR ↑	MT ↑	PT ↓	ML ↓	FP ↓	FN ↓	IDs ↓	FM ↓	MOTA ↑	MOTP ↑	OSPA$^{(2)}$ ↓
YOLOv3+MV-GLMB-OC	**59.8%**	**61.0%**	**60.8%**	**3**	4	**0**	**404**	**951**	**67**	**54**	**60.6%**	**45.0%**	**0.65**
YOLOv3+MV-GLMB-OC*	55.9%	54.9%	57.1%	**3**	3	1	689	1125	80	85	55.3%	43.4%	0.71
YOLOv3+MV-GLMB	49.5%	50.1%	45.0%	**3**	**2**	2	715	1750	94	91	49.3%	42.6%	0.78
Faster-RCNN(VGG16)+MV-GLMB-OC	58.1%	60.8%	59.4%	**3**	4	**0**	451	1008	72	57	59.9%	43.1%	0.66
Faster-RCNN(VGG16)+MV-GLMB-OC*	55.9%	53.6%	51.6%	**3**	3	1	569	1519	81	88	51.4%	42.7%	0.75
Faster-RCNN(VGG16)+MS-GLMB	48.8%	45.3%	41.7%	**3**	3	1	734	1493	96	98	43.3%	43.9%	0.81

CLEAR MOT scores and OSPA$^{(2)}$ 3D errors with 3D GIoU base-distance

Detectors (monocular): Faster-RCNN(VGG16) and YOLOv3

GLMB Filters: Standard Multi-Sensor (MS), Multi-View with Occlusion Model (MV)

Asterisk e.g. MV-GLMB-OC* indicates tracking while reconfiguring cameras

Ong et. al. “A Bayesian Filter for Multi-view 3D Multi-object Tracking with Occlusion Handling,” IEEE Trans. PAMI, 2020.

Application: Autonomous Driving

Autonomous Driving: **SLAM + multi-object filtering**

Prototype system with E-Class Mercedes-Benz

H. Deusch et. al. “The Labeled Multi-Bernoulli SLAM filter,” IEEE Signal Proc. Letters, 22(10), 2015.

Application: Reinforcement Learning

Use drone to autonomously track and localize animals

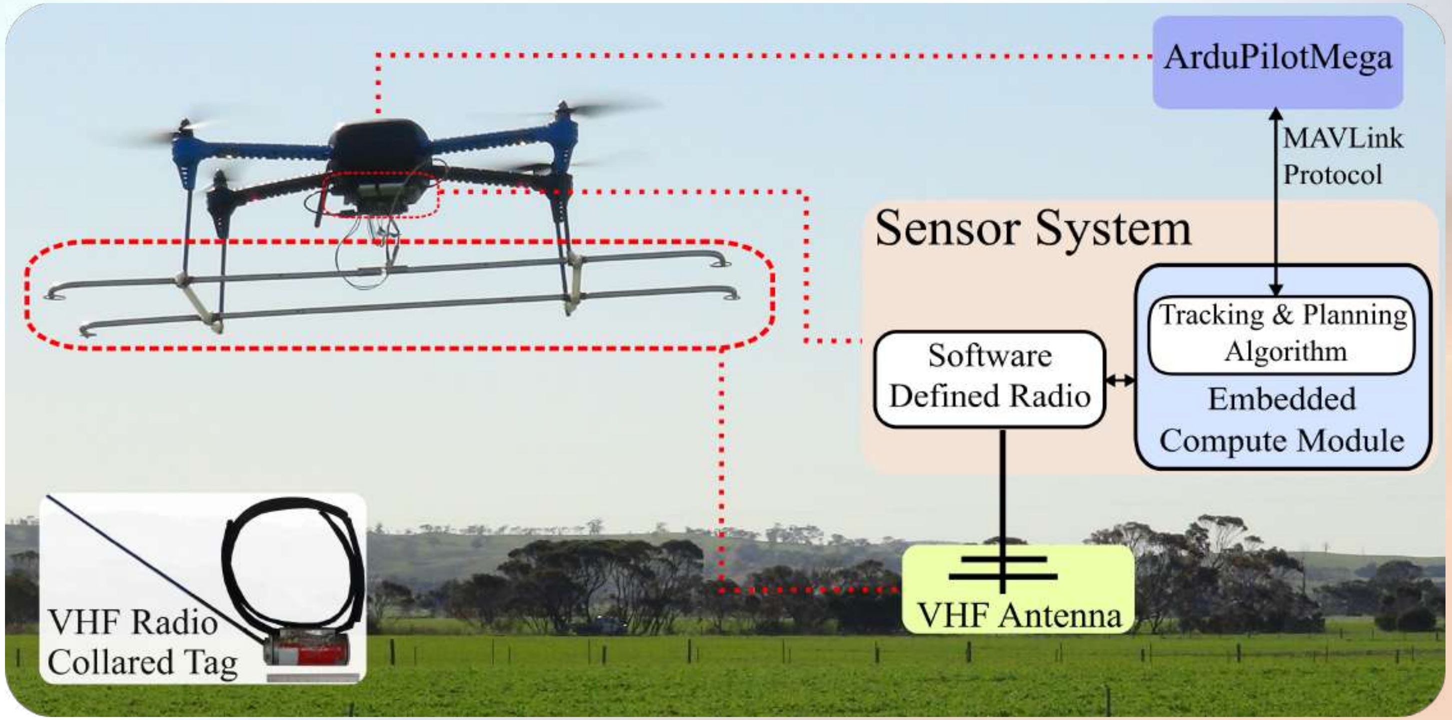
How biologists track VHF-collared wildlife:

- Trek long distances with VHF radio receivers, directional antennas and battery packs
- Manually home in on radio collar signals from collared wildlife to find and locate them

Handheld device

Application: Reinforcement Learning

Autonomous Drone:
- Pros: Low cost, reduces time and operating costs
- Cons: Introduce disturbances to wildlife

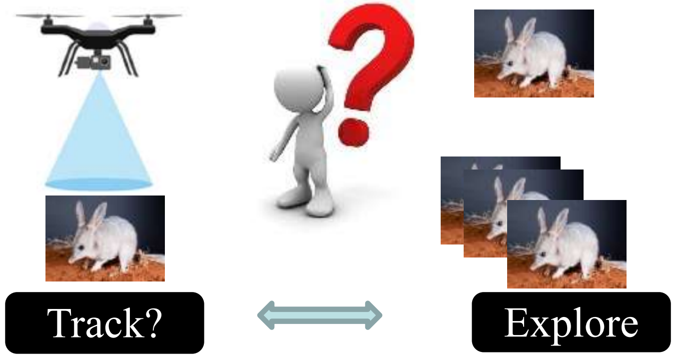


- Need intelligent drones that can perform prescribed mission on their own

Application: Reinforcement Learning

Team of agents to **track** and **search** for a time-varying number of mobile objects

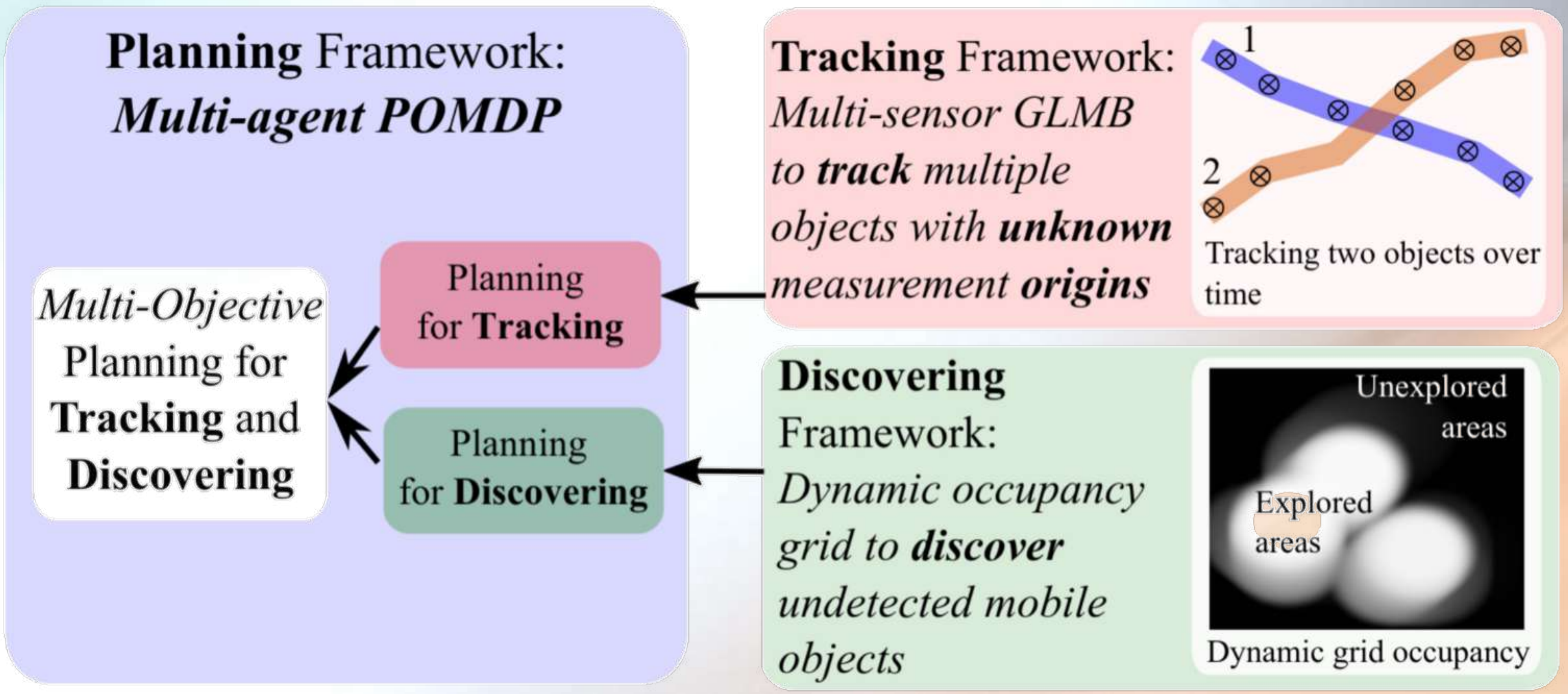
- **Real-time operation**
- **Limited FoVs**
- **Competing Objectives**



Nguyen et al., "Multi-Objective Multi-Agent Planning for Jointly Discovering and Tracking Mobile Objects," AAAI-2020.

Nguyen et al., "Multi-Objective Multi-Agent Planning for Discovering and Tracking Unknown and Varying Number of Mobile Objects," 2022 (arXiv:2203.04551)

Application: Reinforcement Learning



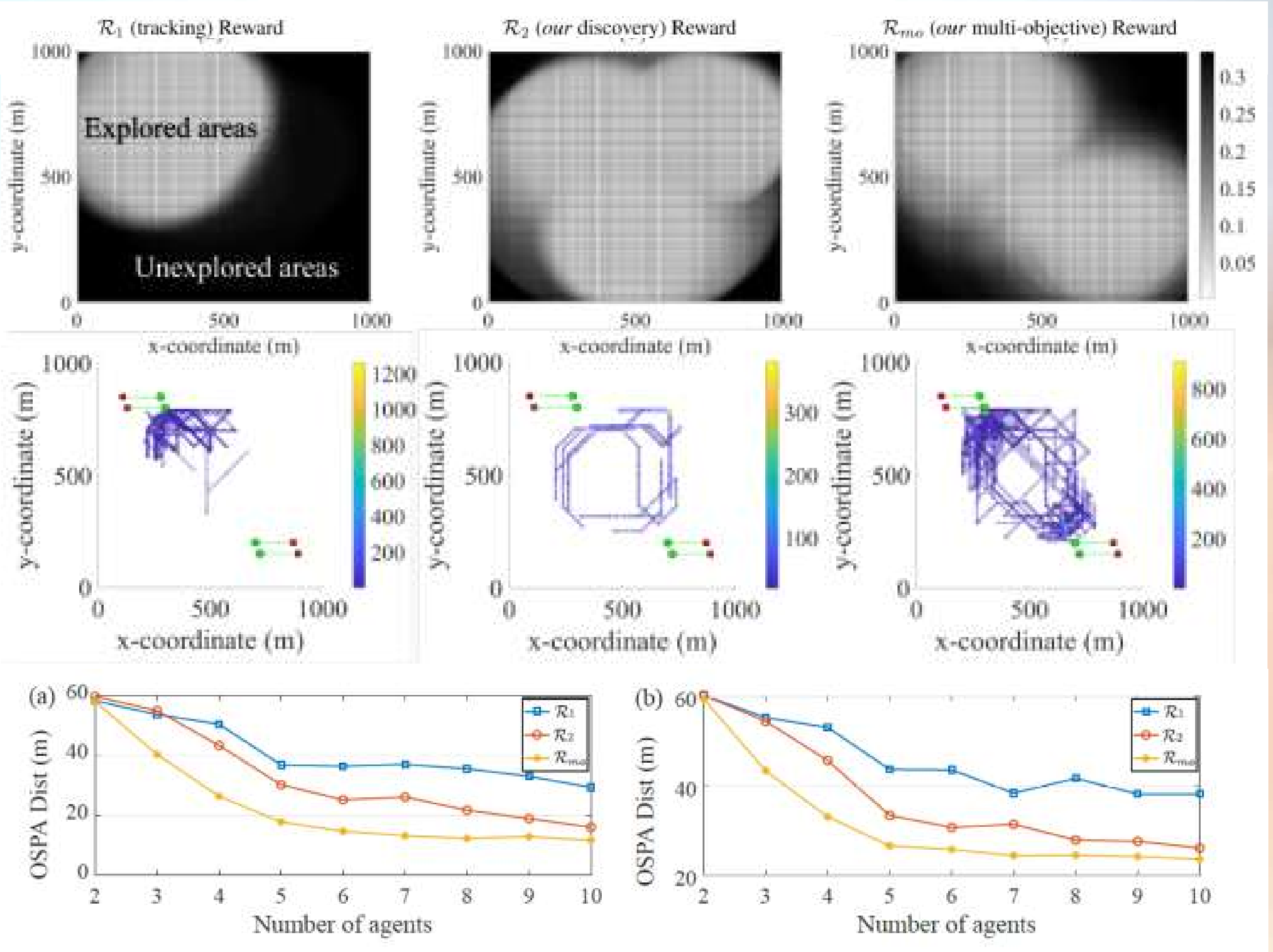
Inspired by the multi-objective RFS control solution in [Zhu et. al. 2019]

Nguyen et al., “Multi-Objective Multi-Agent Planning for Jointly Discovering and Tracking Mobile Objects,” AAAI-2020.

Nguyen et al., “Multi-Objective Multi-Agent Planning for Discovering and Tracking Unknown and Varying Number of Mobile Objects,” 2022 (arXiv:2203.04551)

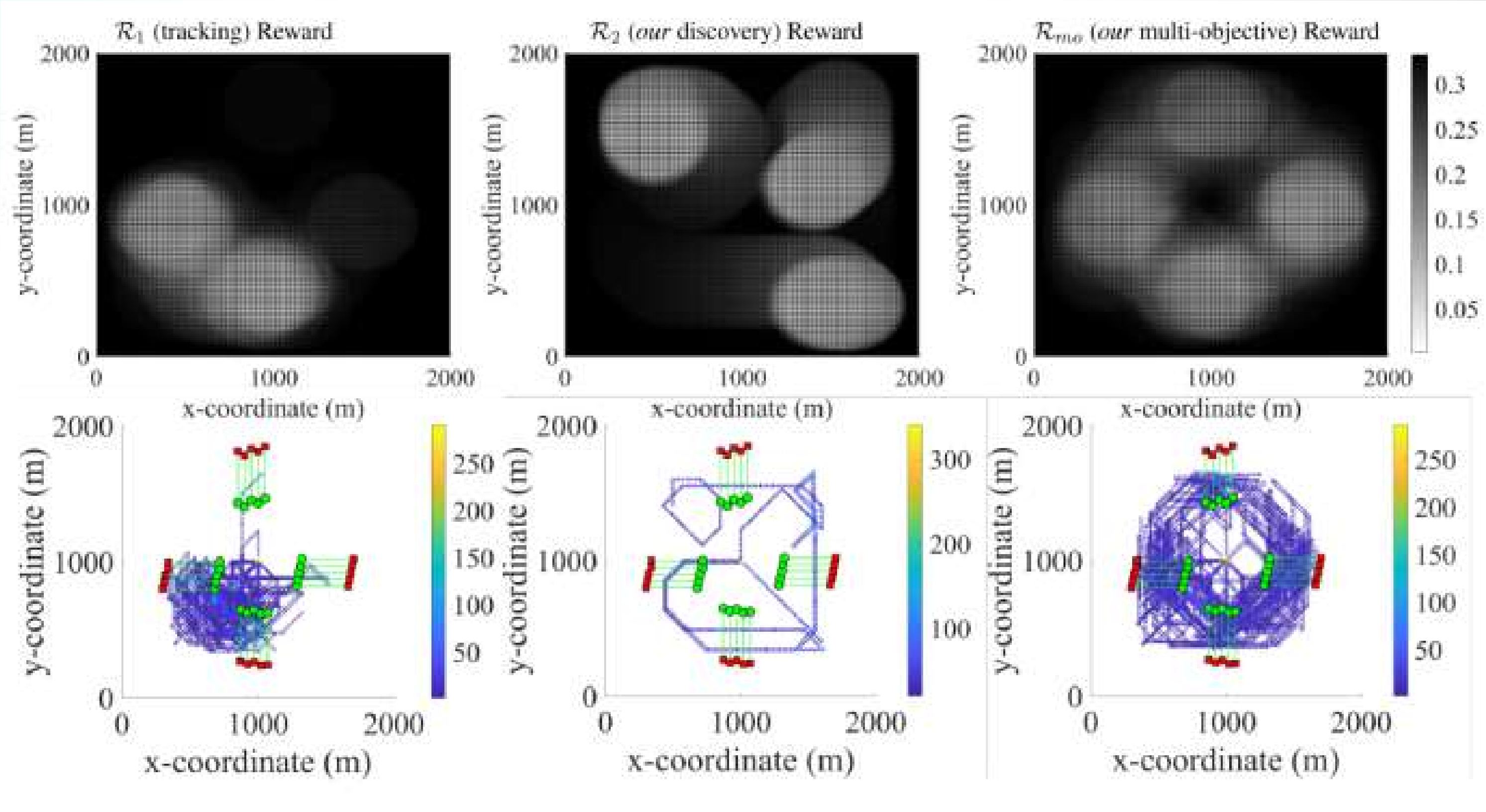
Application: Reinforcement Learning

Multi-objective strategies are more effective …



Application: Reinforcement Learning

Multi-objective strategies are more effective …



Conclusions

Multi-Object HMM provides:

- insights into the foundations of applications involving multiple objects &
- efficient and scalable solutions not possible previously

Many interesting problems in

- Artificial intelligence, machine learning, data mining
- Communications, Astro-dynamics (Space Debris)
- Bio-medical research: cell microscopy, brain imaging ...

Preprints/latest works: http://ba-ngu.vo-au.com/publications.html
Matlab code: http://ba-tuong.vo-au.com/codes.html

Thank You!