



**VIASM**

VIETNAM INSTITUTE FOR  
ADVANCED STUDY IN MATHEMATICS

**Conference on**  
*Commutative Algebra and its interaction  
with Algebraic Geometry and  
Combinatorics 2023*

**PROGRAM AND  
ABSTRACTS**

# Conference on Commutative Algebra and its interaction with Algebraic Geometry and Combinatorics 2023

Vietnam Institute for Advanced Study in Mathematics (VIASM)  
19/06/2023 – 23/06/2023

## Sponsors

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**Location:** Vietnam Institute for Advanced Study in Mathematics  
157 Chua Lang Road, Dong Da Restrict, Ha Noi, Vietnam

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- **Lê Minh Hà** (Vietnam Institute for Advanced Study in Mathematics),  
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- **Lê Tuấn Hoa** (Institute of Mathematics, Hanoi), Co-chair
- **Nguyễn Đăng Hợp** (Institute of Mathematics, Hanoi)
- **Nguyễn Công Minh** (Hanoi University of Science and Technology)

**Schedule of the Conference on**  
*Commutative Algebra and its interaction with Algebraic  
 Geometry and Combinatorics 2023*

**Monday, June 19**

Chair	Le Tuan Hoa	
8h45 – 9h00	Opening ceremony	
9h00 – 9h50	<b>Bernd Ulrich</b>	Generalized Jouanolou duality, weakly Gorenstein rings, and the implicitization problem
9h50 – 10h00	Break	
10h00 – 10h50	<b>Naoki Terai</b>	Level Stanley-Reisner rings with codimension two
10h50 – 11h10	Break	
11h10 – 12h00	<b>Trần Nam Trung</b>	Depth functions of powers of edge ideals of trees and cycles
12h00 – 13h30	LUNCH BREAK	
Chair	Claudia Polini	
13h30 – 14h20	<b>Rosa Miró-Roig</b>	Gröbner's problem
14h20 – 14h30	Break	
14h30 – 15h20	<b>Nguyễn Công Minh</b>	On the sequentially Cohen-Macaulay property of monomial ideals
15h20 – 15h40	Break	
15h40 – 16h30	<b>A.V. Jayanthan</b>	Depth of binomial edge ideals in terms of diameter and graph connectivity

**Tuesday, June 20**

Chair	Ngo Viet Trung	
9h00 – 9h50	<b>Tài Huy Hà</b>	Regularity of graded families of homogeneous ideals
9h50 – 10h00	Break	
10h00 – 10h50	<b>Sara Faridi</b>	Resolutions of powers of square-free monomial ideals (I)
10h50 – 11h10	Break	
11h10 – 12h00	<b>Susan Morey</b>	Resolutions of powers of square-free monomial ideals (II)
12h00 – 13h30	LUNCH BREAK	

Chair	Maria Evelina Rossi	
13h30 – 14h20	<b>Steven Dale Cutkosky</b>	Characterization of deeply ramified fields
14h20 – 14h30	Break	
14h30 – 15h20	<b>Hailong Dao</b>	Fitting ideals of infinite resolutions
15h20 – 15h40	Break	
15h40 – 16h30	<b>Lê Thanh Nhân</b>	Artinian local cohomology modules under flat base changes

### Wednesday June 21

Chair	Nguyen Dang Hop	
9h00 – 9h50	<b>Maria Evelina Rossi</b>	On the rate of generic Gorenstein K-algebras
9h50 – 10h00	Break	
10h00 – 10h50	<b>Giulio Caviglia</b>	Bounds on the number of generators of prime ideals
10h50 – 11h10	Break	
11h10 – 11h35	<b>Nguyễn Thu Hằng</b>	Depth stability of cover ideals
11h35 – 12h00	<b>Văn Đức Trung</b>	Small perturbations of ideal in local rings

### Thursday, June 22

Chair	Naoki Terai	
9h00 – 9h50	<b>Claudia Polini</b>	Generalized multiplicity and integral dependence
9h50 – 10h00	Break	
10h00 – 10h50	<b>Arvind Kumar</b>	Resurgence and asymptotic resurgence numbers of graded families of ideals
10h50 – 11h10	Break	
11h10 – 12h00	<b>Trần Tuấn Nam</b>	Formal generalized local cohomology
12h00 – 13h30	LUNCH BREAK	
Chair	Rosa Miró-Roig	
13h30 – 14h20	<b>Sijong Kwak</b>	Remark on the generalized gonality conjecture of smooth projective curves
14h20 – 14h30	Break	
14h30 – 15h20	<b>Đỗ Trọng Hoàng</b>	Cohen-Macaulayness of graphs arising from a finite commutative ring
15h20 – 15h40	Break	
15h40 – 16h30	<b>Vũ Quang Thanh</b>	Cohen-Macaulay Rees algebras of edge ideals of graphs

19h00 – 21h00	Conference banquet (details to be provided later)
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**Friday, June 23**

Chair	Bernd Ulrich	
9h00 – 9h50	<b>Joan Elias</b>	Additive combinatorics and algebraic geometry
9h50 – 10h00	Break	
10h00 – 10h50	<b>Hema Srinivasan</b>	Gluing operations and some interesting consequences
10h50 – 11h10	Break	
11h10 – 11h35	<b>Trần Quang Hoá</b>	Lefschetz properties of artinian monomial algebras associated to graphs
11h35 – 12h00	<b>Thái Thành Nguyễn</b>	Some invariants of geometrically vertex decomposable ideals
12h00 – 13h30	LUNCH BREAK	
Chair	Tài Huy Hà	
13h30 – 14h20	<b>Marc Chardin</b>	Standard multigraded Tor and local cohomology
14h20 – 14h50	Break	
14h50 – 15h40	<b>Jugal Verma</b>	On a conjecture of Ilya Smirnov about Hilbert-Kunz multiplicity of powers of ideals
15h40 – 16h00	Closing remarks	

# ABSTRACT

## 1. Bounds on the number of generators of prime ideals

Giulio Caviglia (*Purdue University, USA. Email address: gcavigli@purdue.edu*)

Let  $S$  be a polynomial ring over any field  $k$ , and let  $P$  in  $S$  be a non-degenerate homogeneous prime ideal of height  $h$ . When  $k$  is algebraically closed, a classical result attributed to Castelnuovo establishes an upper bound on the number of linearly independent quadrics contained in  $P$  which only depends on  $h$ . We significantly extend this result by proving that the number of minimal generators of  $P$  in any degree  $j$  can be bounded above by an explicit function that only depends on  $j$  and  $h$ . In addition to providing a bound for generators in any degree  $j$ , not just for quadrics, our techniques allow us to drop the assumption that  $k$  is algebraically closed. By means of standard techniques, we also obtain analogous upper bounds on higher graded Betti numbers of any radical ideal.

This is a joint work with Alessandro De Stefani.

## 2. Standard multigraded Tor and local cohomology

Marc Chardin (*Sorbonne University, France. Email address: marc.chardin@imj-prg.fr*)

I will present joint work with Rafael Holanda connecting the support of Tor modules with the support of local cohomology (the one that provides sheaf cohomology on a product of projective spaces). This is intimately linked to results about local cohomologies supported on monomial ideals. Then I will present two consequences, one on linear resolutions of truncations and one on the regularity of powers of ideals.

## 3. Characterization of deeply ramified fields

Steven D. Cutkosky (*University of Missouri USA. Email address: CutkoskyS@missouri.edu*)

We give a simple explicit proof of a theorem by Gabber and Romero that deeply ramified fields are characterized by vanishing of Kahler differentials. Perfectoid fields are an example of deeply ramified fields. Our proof reduces to the case of extensions of valuation rings in Artin-Schreier and Kummer extensions where we give complete and explicit descriptions

of the Kahler differentials of these extensions and characterize when they vanish. The essential difficulty arising in the analysis of ramification in positive and mixed characteristic is that defect may appear, and this is divided into dependent and independent defect. One new result that we obtain is that above a deeply ramified field, only independent defect can appear. This is joint work with Franz-Viktor Kuhlmann and Anna Rzepka.

#### **4. Fitting ideals of infinite resolutions**

Hailong Dao (*Kansas University, USA. Email address: hdao@ku.edu*)

Ideals of minors of differentials in free resolutions play an important role in commutative algebra and algebraic geometry. In this talk, we discuss the asymptotic behavior of the ideals of minors in minimal free resolutions over local rings. We show that surprisingly strong periodic patterns exist for any finitely generated module over broad class of ring: those with high Burch indexes, complete intersections and Golod rings. Explicit computations even for small examples are interesting, and intriguing questions abound. The talk will be based on joint work with David Eisenbud and with Michael Brown-Prashanth Sridhar.

#### **5. Additive combinatorics and algebraic geometry**

Joan Elias (*University of Barcelona, Spain. Email address: elias@ub.edu*)

We present a link between the additive combinatorics of subsets of  $\mathbb{Z}^n$  and the geometry of Veronese varieties. In the case  $n = 1$ , we associate to a finite set of non-negative integers  $A$  a monomial projective curve  $C_A$  such that the Hilbert function of  $C_A$  and the cardinalities of the sumsets  $sA$  agree. Moreover, the singularities of  $C_A$  determine the asymptotic behavior of the function  $s \mapsto |sA|$ . We mainly focus on the case  $n = 1$ , if we have time, we will comment some results for a general integer  $n$ .

#### **6. Resolutions of powers of square-free monomial ideals**

Sara Faridi (*Dalhousie University, Canada. Email address: faridi@mathstat.dal.ca*)

In this talk we discuss how to construct free resolutions of powers of monomial ideals starting from the free resolution of the ideal itself. The Taylor resolution is a free resolution that works for any monomial ideal

and can be constructed from the simplicial chain complex of a simplex. The Taylor resolution is sometimes (though rarely) minimal, so its size is justified as it provides a reasonable upper bound for minimal free resolutions of monomial ideals. However as soon as we square a non-principal ideal, the Taylor resolution will never be minimal, and it gets further from minimal as we take higher powers.

The question motivating this talk is what is a reasonable counterpart to the Taylor resolution for powers of monomial ideals?

Similarly, when do powers of ideals have Scarf resolutions, that is, the smallest possible resolutions?

We will present joint work with Susan Cooper, Sabine El Khoury, Tài Hà, Takayuki Hibi, Sarah Mayes-Tang, Susan Morey, Liana Şega, and Sandra Spiroff.

## 7. Regularity of graded families of homogeneous ideals

Tài Huy Hà (*Tulane University, USA. Email address: tha@tulane.edu*)

It is a celebrated theorem proved independently by Cutkosky, Herzog and Trung and by Kodiyalam that the regularity of powers of a homogeneous ideal in a standard graded algebra over a field is asymptotically a linear function. In this talk, we will discuss various aspects and questions arising from this result. Particularly, we will survey results aiming at understanding this asymptotic linear function of the regularity of powers of a homogeneous ideal. We will also explore the generalization to graded families of homogeneous ideals.

## 8. Depth stability of cover ideals

Nguyễn Thu Hằng (*Thai Nguyen University of Sciences, Vietnam. Email address: hangnt@tnus.edu.vn*)

Let  $R = K[x_1, \dots, x_r]$  be a polynomial ring over a field  $K$ . Let  $G$  be a graph with vertex set  $\{1, \dots, r\}$  and let  $J$  be the cover ideal of  $G$ . We give a sharp bound for the stability index of symbolic depth function  $\text{sdtab}(J)$ . In the case  $G$  is bipartite, it yields a sharp bound for the stability index of depth function  $\text{dtab}(J)$  and this bound is exact if  $G$  is a forest.

Our first main result is as follows.

**Theorem 1.**  $\text{sdtab}(J(G)) \leq (\ell(G) + 1)/2$  for every graph  $G$ .

The next result shows that the bound in Theorem 1 is the true value for  $\text{sdtab}(J(G))$  in the case the graph has a perfect ordered matching.



**Proposition 2.** If a graph  $G$  has a perfect ordered matching  $M$ , then

$$\text{sdtab}(J(G)) = (\ell(G) + 1)/2.$$

If  $G$  is bipartite, then  $J(G)^n = J(G)^{(n)}$  for all  $n \geq 1$  by Theorem 5.1 in J. Herzog, T. Hibi and N. V. Trung (*Symbolic powers of monomial ideals and vertex cover algebras*, Adv. Math. **210**(1) (2007), 304 - 322), so that  $\text{sdtab}(J(G)) = \text{dtab}(J(G))$ . Together with Theorem 1 we obtain:

**Corollary 3.** If  $G$  is a bipartite graph, then  $\text{dtab}(J(G)) \leq (\ell(G) + 1)/2$ .

This is from joint work with Mai Phuoc Binh, Truong Thi Hien and Tran Nam Trung.

## 9. Lefschetz properties of artinian monomial algebras associated to graphs

Trần Quang Hóa (*Hue University of Education, Vietnam. Email address: tranquanghoa@hueuni.edu.vn*)

The strong Lefschetz property for an Artinian graded algebra  $A$  over a field  $\mathbb{k}$  simply says that the multiplication by a general linear form  $\times \ell^s : [A]_i \rightarrow [A]_{i+s}$  has maximal rank in every degree  $i$  and every positive integer  $s$ . In particular, if the multiplication by a general linear form  $\times \ell : [A]_i \rightarrow [A]_{i+1}$  has maximal rank in every degree  $i$ , then  $A$  is said to have the *weak Lefschetz property*. At first glance this might seem to be a simple problem of linear algebra. However, determining which graded Artinian  $\mathbb{k}$ -algebras have the weak and/or strong Lefschetz property is notoriously difficult.

In this talk, I will discuss recent results on the weak/strong Lefschetz properties of Artinian graded algebras  $A_G$  defined as follows: Let  $G = (V, E)$  be a simple graph, with  $V = \{1, 2, \dots, n\}$ . Let  $R = \mathbb{k}[x_1, \dots, x_n]$  be the standard polynomial ring over a field  $\mathbb{k}$ . We define the artinian monomial algebra associated to  $G$

$$A_G = \frac{R}{(x_1^2, \dots, x_n^2) + I_G},$$

where  $I_G = (x_i x_j \mid \{i, j\} \in E) \subset R$  is the edge ideal of  $G$ . We classify some classes of simple graphs  $G$  where  $A_G$  has or fails the weak/strong Lefschetz properties. This is a joint work with Hop D. Nguyen.

## 10. Cohen-Macaulayness of graphs arising from a finite commutative ring

Đỗ Trọng Hoàng (*Hanoi University of Science and Technology, Vietnam.*  
*Email address: hoang.dotrong@hust.edu.vn*)

Let  $R$  be a finite commutative ring with nonzero identity. The unitary Cayley graph (resp. unit graph) of  $R$  is the graph obtained by letting all the elements of  $R$  to be the vertices and defining distinct vertices  $x$  and  $y$  to be adjacent if and only if  $x - y$  (resp.  $x + y$ ) is a unit element of  $R$ . This talk is aimed at examining Cohen–Macaulayness of these graphs. Moreover, in the case  $R = \mathbb{Z}_n$ , we completely characterize Cohen–Macaulayness of the surveyed graphs. Such characterizations provide large classes of Cohen–Macaulay and non Cohen–Macaulay graphs. In addition, some issues pertaining to these graphs, such as regularity and Betti numbers, will also be presented. These stem from a joint work with T. Ashitha, T. Asir, and M. R. Pournaki.

## **11. Depth of binomial edge ideals in terms of diameter and graph connectivity.**

A. V. Jayanthan (*Indian Institute of Technology Madras, India. Email address: jayanav@iitm.ac.in*)

Let  $G$  be a simple connected non-complete graph on  $n$  vertices and  $J_G$  be its binomial edge ideal. It is known that  $d(G) + f(G) \leq \text{depth}(S/J_G) \leq n + 2 - \kappa(G)$ , where  $d(G)$  denotes the diameter,  $f(G)$  denotes the number of simplicial vertices and  $\kappa(G)$  denotes the graph connectivity of  $G$ . Hibi and Saeedi Madani characterized  $G$  such that  $d(G) + f(G) = n + 2 - \kappa(G)$ . In this article, we characterize graphs  $G$  such that  $d(G) + f(G) + 1 = n + 2 - \kappa(G)$  and compute the depth of  $S/J_G$  for all such  $G$ .

## **12. Resurgence and asymptotic resurgence numbers of graded families of ideals**

Arvind Kumar (*Chennai Mathematical Institute, India. Email address: arvokumar11@gmail.com*)

This is based on joint works with Tài Huy Hà, Hop D. Nguyen, Thai Thanh Nguyen. We define the resurgence and asymptotic resurgence numbers associated to a pair of graded families of ideals in a Noetherian ring. These notions generalize the well-studied resurgence and asymptotic resurgence of an ideal in a polynomial ring. We examine when these invariant are finite and rational. We investigate situations where these invariant

can be computed via Rees valuations. We study how the asymptotic resurgence changes when a family is replaced by its integral closure. Many examples are given to show that known properties of resurgence and asymptotic resurgence of an ideal do not generally extend to that of a pair of graded families of ideals.

### **13. Remark on the generalized gonality conjecture of smooth projective curves**

Sijong Kwak (*Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea. Email address: sjkwak@kaist.ac.kr*)

The well-known gonality conjecture of smooth curves was raised by Green-Lazarsfeld and proved by Ein-Lazarsfeld in 2016. In this talk, I'd like to consider this conjecture in a category of the higher secant varieties of a curve and investigate the Betti table of the higher secant varieties of a curve. It would be very interesting to connect the vanishing and non-vanishing of their Betti numbers to the gonality sequence of a curve. This is an on-going work with Junho Choe and Jinhyung Park.

### **14. On the sequentially Cohen-Macaulay property of monomial ideals**

Nguyễn Công Minh (*Hanoi University of Science and Technology, Vietnam. Email address: minh.nguyencong@hust.edu.vn*)

The notion of sequentially Cohen-Macaulay was introduced by R. Stanley as a generalization of the Cohen-Macaulay property in connection with the work of Björner and Wachs on nonpure shellability. In this talk, we will discuss the problem of characterizing the sequentially Cohen-Macaulay property of a certain class of monomial ideals in terms of their combinatorial invariants. These are based on joint work with Hoang Le Truong and with Ly Thi Kieu Diem and Thanh Vu.

### **15. Gröbner's problem**

Rosa Maria Miró-Roig (*University of Barcelona, Spain. Email address: miro@ub.edu*)

In my talk, I will address Gröbner's problem. It is a longstanding open problem in commutative algebra and algebraic geometry, posed by W.

Gröbner in 1969 and it aims to determine whether a (monomial) projection of a Veronese variety is an arithmetically Cohen-Macaulay variety. I will summarize what is known about this problem and explain some recent contributions.

## 16. Resolutions of powers of square-free monomial ideals

Susan Morey (*Texas State University-San Marcos, USA. Email address: morey@txstate.edu*)

Using combinatorial structures to obtain resolutions of monomial ideals traces back to Diana Taylor's thesis, where a simplex associated to the generators of a monomial ideal was used to construct a free resolution of the ideal. This concept has been expanded, with various authors determining conditions under which simplicial or cellular complexes can be associated to monomial ideals to produce a free resolution. This talk will examine structures that produce minimal free resolutions of powers of square-free monomial ideals. The properties of the original monomial ideal generally determine which structures are optimal. This talk will examine powers of square-free monomial ideals of projective dimension one. Faridi and Hersey proved that a monomial ideal has projective dimension one if and only if there is an associated tree (one dimensional acyclic simplicial complex) that supports a free resolution of the ideal. The talk will show how, for each power  $r > 1$ , to use the tree associated to a square-free monomial ideal  $I$  of projective dimension one to produce a cellular complex that supports a free resolution of  $I^r$ . Moreover, each of these resolutions will be minimal resolutions. These cellular resolutions can also be viewed as strands of the resolution of the Rees algebra of  $I$ . How the results extend to the case of ideals of projective dimension two will also be explored. This work is joint with Susan Cooper, Sabine El Khoury, Sara Faridi, Sarah Mayes-Tang, Liana Şega, and Sandra Spiroff and began as part of a research project initiated at a BIRS workshop "Women in Commutative Algebra" in Fall 2019.

## 17. Formal generalized local cohomology

Trần Tuấn Nam (*Ho Chi Minh City University of Education, HCM City, Vietnam. Email address: namtuantran@gmail.com*)

Throughout this talk,  $(R, \mathfrak{m})$  is a Noetherian local commutative ring with the non-zero identity. It is well-known that Grothendieck's local co-

homology is an important tool in commutative algebra and algebraic geometry. In 1970 Herzog introduced the concept of generalized local cohomology which was a generalization of Grothendieck's local cohomology. Let  $I$  be an ideal of  $R$  and  $M, N$   $R$ -modules, the  $t$ th generalized local cohomology module  $H_I^i(M, N)$  of  $M, N$  with respect to  $I$  was defined by

$$H_I^i(M, N) = \varinjlim_t \text{Ext}^i(M/I^t M, N).$$

Then in 2007, P. Schenzel studied the modules of the form

$$\mathcal{F}_I^i(M) = \varprojlim_t H_m^i(M/I^t M)$$

and called them formal local cohomology modules.

The purpose of our work is to introduce a concept of formal generalized local homology modules which is a generalization of P. Schenzel's concept of formal local cohomology modules and study its basic properties. The  $i$ th  $I$ -formal generalized local cohomology module of  $M, N$  with respect to  $I$  is defined by

$$\mathcal{F}_I^i(M, N) = \varprojlim_t H_m^i(M, N/I^t N).$$

Note that, when  $M = R$  the formal generalized local cohomology modules coincide with the formal local cohomology modules. Moreover, if  $\text{Supp}(N) \subseteq V(I)$  then  $\mathcal{F}_I^i(M, N) \cong H_m^i(M, N)$  the generalized local cohomology modules.

The vanishing and non-vanishing, the Artinianness of the formal generalized local cohomology modules will be shown. We also study some properties of the top formal generalized local homology modules.

## 18. Artinian local cohomology modules under flat base changes

Lê Thanh Nhân (*Ministry of Education and Training, Vietnam. Email address: nhanlt2014@gmail.com*)

This is a joint work with T. D. M. Chau. Let  $\varphi : R \rightarrow S$  be a flat homomorphism between commutative Noetherian rings. The module structures under the effect of  $\varphi$  have attracted the interest of mathematicians. H-B. Foxby (Proc. AMS, 1975) and H-B. Foxby, A. Thorup (Proc. AMS, 1977) have studied injective modules and minimal injective resolutions under

flat base changes. The Noetherian module structures between  $R$  and  $S$  have been investigated by A. J. Frankild, S. Sather-Wagstaff, R. Wiegand (Michigan Math. J., 2008) and B. J. Anderson, S. Sather-Wagstaff (Proc. AMS., 2014). Specially, for a finitely generated  $R$ -module  $M$ , we have the following well-known shifted principle for associated primes between  $R$  and  $S$

$$\text{Ass}_S(M \otimes_R S) = \bigcup_{\mathfrak{p} \in \text{Ass}_R(M)} \text{Ass}(S/\mathfrak{p}S).$$

and if  $\varphi$  is faithfully flat, then the inverse shifted principle holds true

$$\text{Ass}_R(M) = \{\varphi^{-1}(\mathfrak{P}) \mid \mathfrak{P} \in \text{Ass}_S(M \otimes_R S)\}.$$

On the other hand, I. G. Macdonald<sup>1</sup> defined the set of attached primes  $\text{Att}_R(A)$  of an Artinian  $R$ -module  $A$ , which makes an important role similar to that of the set of associated primes of finitely generated modules. So, it is natural to ask about an analogy of the above shifted principles for Artinian modules. In case where  $S = R_{\mathfrak{p}}$  with  $\mathfrak{p} \in \text{Spec}(R)$ , R. Y. Sharp<sup>2</sup> proved that if the local ring  $(R, \mathfrak{m})$  is a quotient of a Gorenstein local ring, then the shifted localization principle holds true for all Artinian local cohomology module  $H_{\mathfrak{m}}^i(M)$

$$\text{Att}_{R_{\mathfrak{p}}} H_{\mathfrak{p}R_{\mathfrak{p}}}^{i-\dim(R/\mathfrak{p})}(M_{\mathfrak{p}}) = \{\mathfrak{q}R_{\mathfrak{p}} \mid \mathfrak{q} \in \text{Att}_R \widehat{H}_{\mathfrak{m}}^i(M), \mathfrak{q} \subseteq \mathfrak{p}\}.$$

In case where  $S = R_{\mathfrak{p}}$  or  $S = \widehat{R}$ , Quy-Nhan<sup>3</sup> improved the result of R. Y. Sharp as follows: The local ring  $(R, \mathfrak{m})$  is a quotient of a Cohen-Macaulay local ring if and only if the above shifted localization principle holds true for all  $H_{\mathfrak{m}}^i(M)$ , if and only if the following shifted completion principle holds true for all  $H_{\mathfrak{m}}^i(M)$

$$\text{Att}_{\widehat{R}} H_{\mathfrak{m}}^i(M) = \bigcup_{\mathfrak{p} \in \text{Att}_R H_{\mathfrak{m}}^i(M)} \text{Ass}(\widehat{R}/\mathfrak{p}\widehat{R}).$$

If  $d = \dim_R(M)$  and  $I$  is an ideal of the local ring  $(R, \mathfrak{m})$ , then the shifted principles under localization and completion for attached primes of the Artinian local cohomology module  $H_I^d(M)$  have been established by Chau-Nga-Nhan (Acta Math. Vietnam., 2022).

<sup>1</sup>I. G. Macdonald, *Secondary representation of modules over a commutative ring*, Symposia Mathematica, **11** (1973), 23-43.

<sup>2</sup>R. Y. Sharp, *Some results on the vanishing of local cohomology modules*, Proc. London Math. Soc., **30** (1975), 177-195.

<sup>3</sup>L. T. Nhan, P. H. Quy, *Attached primes of local cohomology modules under localization and completion*, J. Algebra, **420** (2014), 475-485.

In this talk, we present our recent results about Artinian module structures and shifted principles of attached primes of Artinian local cohomology modules under the effect of an arbitrary flat local homomorphism  $\varphi : (R, \mathfrak{m}) \rightarrow (S, \mathfrak{n})$ .

## 19. Some invariants of geometrically vertex decomposable ideals

Thái Thành Nguyễn (*McMaster University, Canada. Email address: nguyt161@mcmaster.ca*)

Geometric vertex decomposition has been an increasingly useful technique in various algebro-geometric contexts such as liaison theory and Gröbner bases theory. It can be thought as an ideal-theoretic generalization of a vertex decomposition of a simplicial complex. It was shown in the work of Klein-Rajchgot that geometrically vertex decomposable (gvd) ideals possess many nice algebraic properties as those of the Stanley-Reisner ideal of vertex decomposable simplicial complexes. In this talk, we shall discuss some homological invariants of gvd ideals, with emphasis on toric ideals of graphs. The talk will include results from our ongoing project with Jenna Rajchgot and Adam Van Tuyl.

## 20. Generalized multiplicity and integral dependence

Claudia Polini (*University of Notre Dame, USA. Email address: cpolini@nd.edu*)

I will describe some multiplicity based criteria for the integral dependence of ideals and their connections to equisingularity theory and intersection theory. The first numerical criterion for integral dependence of zero dimensional ideals was proved by Rees in 1960 using Hilbert-Samuel multiplicity. Criteria for arbitrary ideals require generalized notion of multiplicities. In this talk I will mainly survey joint work with Ngo Viet Trung, Bernd Ulrich, and Javid Validashti.

## 21. On the rate of generic Gorenstein $K$ -algebras

Maria Evelina Rossi (*University of Genoa, Italy. Email address: rossim@dima.unige.it*)

The rate of a standard graded  $K$ -algebra  $A$  is a measure of the growth

of the shifts in a minimal free resolution of  $K$  as  $A$ -module. In particular  $A$  has rate one if and only if  $A$  is Koszul. It is known that a generic Artinian Gorenstein algebra of socle degree three is Koszul. We extend this result proving that a generic Artinian Gorenstein algebra of socle degree  $s \geq 3$  has rate  $\lfloor \frac{s}{2} \rfloor$ . In the process we need to prove that a generic Artinian Gorenstein  $K$ -algebra of embedding dimension at least four and socle degree  $s \geq 3$  is generated in degree  $\lfloor \frac{s}{2} \rfloor + 1$ . This gives a partial positive answer to a longstanding conjecture stated by M. Boij on the resolution of a generic Artinian Gorenstein ring of odd socle degree. This is a joint work with M. Boij, E. De Negri and A. De Stefani.

## 22. Gluing operations and some interesting consequences

Hema Srinivasan (*University of Missouri, USA. Email address: SrinivasanH@missouri.edu*)

A semigroup  $\langle C \rangle$  in  $\mathbb{N}^n$  is a gluing of  $\langle A \rangle$  and  $\langle B \rangle$  if the set of minimal-generators of  $C$  splits into two parts,  $C = k_1A \sqcup k_2B$  with  $k_1, k_2 \geq 1$ , and the defining ideals of the corresponding semigroup rings are such that  $I_C$  is generated by  $I_A + I_B$  and one special binomial straddling both  $A$  and  $B$ . Two semigroups  $\langle A \rangle$  and  $\langle B \rangle$  can be glued if there exist positive integers  $k_1, k_2$  such that, for  $C = k_1A \sqcup k_2B$ ,  $\langle C \rangle$  is a gluing of  $\langle A \rangle$  and  $\langle B \rangle$ . We denote this by  $C = k_1A \bowtie k_2B$ . In this talk, we will give some necessary and sufficient conditions on  $A$  and  $B$  for the existence of a gluing of  $\langle A \rangle$  and  $\langle B \rangle$  as in “Gluing semigroups: when and how”, *Semigroup Forum*, 101 (2020), 603-618 and “On gluing semigroups in  $\mathbb{N}^n$  and the consequences”, *Res. Math. Sci.* (2022) 9 (2) 1-14. These generalize and explain the previous known results on existence of gluing.

Gluing of semigroups is known to preserve many properties such as Cohen-Macaulay, Gorenstein, Complete intersections with formulae for regularity and Hilbert Functions. We will prove that that if  $A$  and  $B$  are Wilf semigroups, so is their gluing  $A \bowtie B$ .

## 23. Level Stanley-Reisner rings with codimension two

Naoki Terai (*Okayama University, Japan. Email address: terai@okayama-u.ac.jp*)

It is known that 2-Cohen-Macaulay property is equivalent with level property with  $a$ -invariant 0. For Stanley-Reisner rings with codimension two, 2-Cohen-Macaulay and matroid properties are equivalent. We will



talk about their Hilbert functions and conjectures of Stanley and Chari on matroid complexes in the case of codimension two. This talk is based on a joint work with M.R. Pournaki, M. Poursoltani and S. Yassemi.

## 24. Cohen-Macaulay Rees algebras of edge ideals of graphs

Vũ Quang Thanh (Thanh Vu) (*Hanoi University of Science and Technology, Vietnam. Email address: vuqthanh@gmail.com*)

I will discuss the problem of classifying Cohen-Macaulay Rees algebras of edge ideals of graphs. Then I will talk about computing the regularity of Cohen-Macaulay Rees algebras of edge ideals of graphs in terms of its combinatorial invariants. This is based on joint work with Cao Huy Linh and Tran Quang Hoa.

## 25. Depth functions of powers of edge ideals of trees and cycles

Trần Nam Trung (*Institute of Mathematics, VAST, Vietnam. Email address: tntrung@math.ac.vn*)

Let  $S = K[x_1, \dots, x_n]$  be a polynomial ring over a field  $K$ . Let  $I$  be the edge ideal of a graph  $G$  with vertex set  $\{x_1, \dots, x_n\}$ . We study the depth function  $\text{depth}(S/I^t)$ . We prove that

$$\text{depth}(S/I^t) = \min \left\{ \left\lceil \frac{n-t+1}{3} \right\rceil, 1 \right\}$$

if  $G$  is a path of  $n$  vertices.

In the case  $G$  is a cycle of length  $n \geq 5$ , we show that

$$\text{depth}(S/I^t) = \begin{cases} \left\lceil \frac{n-t+1}{3} \right\rceil, & \text{if } 2 \leq t < \lceil (n+1)/2 \rceil, \\ 1, & \text{if } n \text{ is even and } t \geq \lceil (n+1)/2 \rceil, \\ 0, & \text{if } n \text{ is odd and } t \geq \lceil (n+1)/2 \rceil. \end{cases}$$

(Joint work with Nguyen Cong Minh and Thanh Vu.)

## 26. Small perturbations of ideal in local rings

Văn Đức Trung (*Hue University of Education, Vietnam. Email address: vanductrung1985@gmail.com*)

Let  $(R, \mathfrak{m})$  be a Noetherian local ring and  $I$  an ideal of  $R$ . In this talk,

we present the results on the preservation of some invariants of  $R/I$  under small perturbations of  $I$ .

## **27. Generalized Jouanolou duality, weakly Gorenstein rings, and the implicitization problem**

Bernd Ulrich (*Purdue University, USA. Email address: ulrich@math.purdue.edu*)

This talk is concerned with a classical problem in elimination theory, the determination of the implicit equations defining the graphs and images of rational maps between projective varieties. The problem amounts to identifying the torsion in the symmetric algebra of an ideal, and one technique to achieve this is based on a duality statement due to Jouanolou that expresses the torsion of a graded algebra in terms of a graded dual of this algebra. Unfortunately, Jouanolou duality requires the algebra to be Gorenstein, a rather restrictive hypothesis for symmetric algebras. In this talk, I will introduce a generalized notion of Gorensteinness, which we call weakly Gorenstein, and explain how Jouanolou duality can be extended to this larger class of algebras. The generalized duality leads to the solution of the implicitization problem for new classes of rational maps, and can be used in some cases to relate the implicitization problem for an ideal to the implicitization problem for one of its Fitting ideals. The talk is based on joint work with Yairon Cid-Ruiz, Claudia Polini, and Matthew Weaver.

## **28. On a conjecture of Ilya Smirnov about Hilbert-Kunz multiplicity of powers of ideals**

Jugal Verma (*Indian Institute of Technology, Mumbai, India. Email address: verma.jugal@gmail.com*)

I. Smirnov conjectured that the HK multiplicity of large powers of an  $\mathfrak{m}$ -primary ideal  $I$  in Noetherian local ring  $R$  can be expressed in terms of the limits of the Hilbert coefficients of the Frobenius powers of the same ideal. We will present a partial solution of the conjecture under the assumption that the associated graded rings of Frobenius powers of  $I$  have depth at least  $\dim R - 1$ .

## PARTICIPANTS

1. Tran Nguyen An, Thai Nguyen University of Education, Vietnam, antn@tnue.edu.vn
2. Douglas Barnes, Trinity College, University of Cambridge, UK, db875@cam.ac.uk
3. Dư Thị Hòa Bình, Hanoi University, Vietnam, binhdth@hanu.edu.vn
4. Giulio Caviglia, Purdue University, USA, gcavigli@purdue.edu
5. Marc Chardin, Sorbonne University, France, marc.chardin@imj-prg.fr
6. Trần Đỗ Minh Châu, Thai Nguyen University of Education, Vietnam, chautdm@tnue.edu.vn
7. Steven D. Cutkosky, University of Missouri, USA, CutkoskyS@missouri.edu
8. Đoàn Trung Cường, Institute of Mathematics, VAST, Vietnam, dtcuong@math.ac.vn
9. Nguyễn Tự Cường, Institute of Mathematics, VAST, Vietnam, ntcuong@math.ac.vn
10. Hailong Dao, Kansas University, USA, hdao@ku.edu
11. Deblina Dey, Indian Institute Of Technology, Madras, India, ma20d750@smail.iitm.ac.in
12. Lê Xuân Dũng, Hong Duc University, Vietnam, lxdung27@gmail.com
13. Phan Nhật Duy, The Hong Kong University of Science and Technology, Hong Kong, dphan.math@gmail.com
14. Trần Đình Khánh Dương, Michigan State University, USA, tdkduong@gmail.com

15. Nguyễn Minh Đức, Vietnam National University, Vietnam,  
nguyenminhduc\_t64@hus.edu.vn
16. Joan Elias, University of Barcelona, Spain,  
elias@ub.edu
17. Sara Faridi, Dalhousie University, Canada,  
faridi@mathstat.dal.ca
18. Tài Huy Hà, Tulane University, USA,  
tha@tulane.edu
19. Le Minh Ha, Vietnam Institute for Advanced Study in Mathematics  
(VIASM), Vietnam,  
leminhha@viasm.edu.vn
20. Nguyễn Thị Ánh Hằng, Thai Nguyen University of Education, Viet-  
nam,  
hangnthianh@gmail.com
21. Nguyễn Thu Hằng, Thai Nguyen University of Sciences, Vietnam,  
hangnt@tnus.edu.vn
22. Trương Thị Hiền, Hong Duc University, Vietnam,  
hientruong86@gmail.com
23. Hà Thị Thu Hiền, Foreign Trade University, Vietnam,  
thuhienha504@gmail.com
24. Le Tuan Hoa, Institute of Mathematics, VAST, Vietnam,  
lthoa@math.ac.vn
25. Trần Quang Hóa, Hue University, Vietnam,  
tranquanghoa@hueuni.edu.vn
26. Đỗ Trọng Hoàng, Hanoi University of Science and Technology, Viet-  
nam,  
hoang.dotrong@hust.edu.vn
27. Nguyễn Văn Hoàng, University of Transport and Communications,  
Vietnam  
hoangnv@utc.edu.vn
28. Liêu Long Hồ, Institute of Mathematics, VAST, Vietnam,  
lieulongho@gmail.com

29. Lê Xuân Hùng, Hanoi University of Natural Resources and Environment, Vietnam,  
lxhung@hunre.edu.vn
30. Dương Thị Hương, Thang Long University, Vietnam,  
duonghuongtlu@gmail.com
31. A. V. Jayanthan, Indian Institute of Technology Madras, India,  
jayanav@iitm.ac.in
32. Nguyễn Trung Kiên, Phan Huy Chu High School, Vietnam,  
hoctoantkqo@gmail.com
33. Do Van Kien, Hanoi Pedagogical University 2, Vietnam,  
dovankien@hpu2.edu.vn
34. Ajay Kumar, Indian Institute of Technology, Jammu, India,  
ajay.kumar@iitjammu.ac.in
35. Arvind Kumar, Chennai Mathematical Institute, India,  
arvkumar11@gmail.com
36. Rajiv Kumar, Indian Institute of Technology, Jammu. India,  
rajiv.kumar@iitjammu.ac.in
37. Sijong Kwak, Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea,  
sjkwak@kaist.ac.kr
38. Hà Minh Lam, Institute of Mathematics, VAST, Vietnam,  
hmlam@math.ac.vn
39. Nguyễn Thường Lạng, National Economics University, Vietnam,  
langnt@neu.edu.vn
40. Trần Ngọc Lễ, New Mexico State University, USA,  
letran95@nmsu.edu
41. Cao Huy Linh, Hue University of Education, Vietnam,  
caohuylinh@hueuni.edu.vn
42. Đồng Hữu Mậu, Hanoi Metropolitan University, Vietnam,  
dhmau@daihocthudo.edu.vn

43. Nguyễn Công Minh, Hanoi University of Sciences and Technology, Vietnam,  
minh.nguyencong@hust.edu.vn
44. Vũ Quang Minh, University of Science and Technology of Hanoi, Vietnam,  
minhvuq.ba12-123@st.usth.edu.vn
45. Rosa Maria Miró-Roig, University of Barcelona, Spain,  
miro@ub.edu
46. Susan Morey, Texas State University-San Marcos, USA,  
morey@txstate.edu
47. Nguyễn Đình Nam, Ha Tinh University, Vietnam,  
nam.nguyendinh@htu.edu.vn
48. Phạm Hồng Nam, Thai Nguyen University of Sciences, Vietnam,  
namph@tnus.edu.vn
49. Trần Tuấn Nam, Ho Chi Minh City University of Education, Vietnam,  
namtuantran@gmail.com
50. Nguyễn Văn Nghĩa, Hung Vuong University, Vietnam,  
nguyenvannghia@hvu.edu.vn
51. Đặng Minh Ngọc, Vietnam National University, Vietnam,  
bright.jewel.2016@gmail.com
52. Hop D. Nguyen, Institute of Mathematics, VAST, Vietnam,  
ndhop@math.ac.vn
53. Thái Thành Nguyễn, McMaster University, Canada,  
nguyt161@mcmaster.ca
54. Lê Thanh Nhân, Ministry of Education and Training, Vietnam,  
nhanlt2014@gmail.com
55. Nguyễn Thị Kiều Nhung, National Economics University, Vietnam,  
nhungntk.yec20@gmail.com
56. Thiều Đình Phong, Vinh University, Vietnam,  
phongtd@vinhuni.edu.vn
57. Claudia Polini, University of Notre Dame, USA,  
cpolini@nd.edu

58. Bùi Quốc, Institute of Mathematics, VAST, Vietnam,  
buiquoc\_t62@hus.edu.vn
59. Phạm Hùng Quý, FPT University, Hanoi, Vietnam,  
quyph@fe.edu.vn
60. Maria Evelina Rossi, University of Genoa, Italy,  
rossim@dima.unige.it
61. Kamalesh Saha, Indian Institute of Technology Gandhinagar, India,  
kamalesh.saha44@gmail.com
62. Hema Srinivasan, University of Missouri, USA,  
SrinivasanH@missouri.edu
63. Nguyễn Thị Thanh Tâm, Hung Vuong University, Vietnam,  
thanhtamnguyenhv@gmail.com
64. Naoki Terai, Okayama University, Japan,  
terai@okayama-u.ac.jp
65. Vũ Quang Thanh (Thanh Vu), Hanoi University of Science and Technology, Vietnam,  
vuqthanh@gmail.com
66. Lưu Phương Thảo, Thai Nguyen University of Education, Vietnam,  
thaolp@tnue.edu.vn
67. Đào Văn Thịnh, Institute of Mathematics, VAST, Vietnam,  
daothinh1812@gmail.com
68. Phan Thị Thủy, Hanoi National University of Education, Vietnam,  
phanthuy@hnue.edu.vn
69. Đoàn Quang Tiến, Viet Nam National University Ho Chi Minh City, Vietnam,  
doanquangtien1442001@gmail.com
70. Nguyễn Thị Trà, Hanoi Pedagogical University 2, Vietnam,  
nguyentra.bsu@gmail.com
71. Nguyễn Quang Tri, School of Information & Communication Technology - Hanoi University of Science and Technology, Vietnam,  
nqt068@gmail.com

72. Văn Đức Trung, Hue University of Education, Vietnam,  
vanductrung1985@gmail.com
73. Tran Nam Trung, Institute of Mathematics, VAST, Vietnam,  
tntrung@math.ac.vn
74. Đinh Thành Trung, FPT University, Hanoi, Vietnam,  
trung.dinh.nb@gmail.com
75. Ngo Viet Trung, Institute of Mathematics, VAST, Vietnam,  
nvtrung@math.ac.vn
76. Hoang Le Truong, Institute of Mathematics, VAST, Vietnam,  
hltruong@math.ac.vn
77. Hoàng Tùng, FPT University, Hanoi, Vietnam  
hoangtungkhtn@gmail.com
78. Bernd Ulrich, Purdue University, USA,  
ulrich@math.purdue.edu
79. Dharm Veer, Indian Institute of Technology Gandhinagar, India,  
dharm.v@iitgn.ac.in
80. Jugal Verma, Indian Institute of Technology, Mumbai, India,  
verma.jugal@gmail.com
81. Đỗ Hoàng Việt, Institute of Mathematics, VAST, Vietnam,  
vietdohoang10@gmail.com
82. Anoot Kumar Yadav, Indian Institute of Technology Patna, India  
anoot\_2021ma06@iitp.ac.in
83. Hoang Ngoc Yen, Institute of Mathematics, VAST, Vietnam,  
hnyen91@gmail.com



**Wifi: VIASM**  
**Pass: viasm157@2021**