VIASM Summer School in Mathematical Physics 2024

Hue University of Education, August 5-10, 2024

August 5, 2024

8:00-8:15	**Registration**
8:15-8:30	**Opening**
8:30-9:30	**Robert Seiringer** (IST Austria) Mathematical Foundations of Density Functional Theory (I)
9:30-10:00	**Coffee Break**
10:00-11:00	**Robert Seiringer** (IST Austria) Mathematical Foundations of Density Functional Theory (II)
11:15-12:15	**Discussion**
12:00-14:00	**Lunch Break**
14:00-15:00	**Tô Tất Đạt** (Sorbonne Université) The Phi43 measure on compact Riemannian 3-manifolds
15:00-15:30	**Coffee Break**
15:30-16:30	**Maxime Van De Moortel** (Rutgers University) Strong Cosmic Censorship Conjecture and black hole dynamics

August 6, 2024

8:00-9:00	**Mitia Duerinckx** (Université Libre de Bruxelles) New methods for mean-field limits of classical particle systems (I)
9:00-9:30	**Coffee Break**
9:30-10:30	**Mitia Duerinckx** (Université Libre de Bruxelles) New methods for mean-field limits of classical particle systems (II)
10:30-11:00	**Coffee Break**
11:00-12:00	**Discussion**
12:00-14:00	**Lunch Break**
14:00-15:00	**Jia Shi** (Massachusetts Institute of Technology) Non-radial implosion for compressible Euler and Navier-Stokes in T3 and R3
15:00-15:30	**Coffee Break**
15:30-15:45	**Chanjin You** (Penn State) Phase mixing estimates for the nonlinear Hartree equation of infinite rank
15:45-16:00	**Vũ Hồ Thảo Thuận** (Monash University) Hamiltonian approach to 2-layer dispersive stratified fluids
16:00-16:15	**Võ Hoàng Hưng** (Saigon University) Liouville Theorems for infinity Laplacian with gradient and KPP type equation
16:15-16:30	**Nguyễn Tiến Tài** (VNU University of Science, Hanoi) Nonlinear Rayleigh-Taylor instability in fluid mechanics and beyond
16:30-16:45	**Jinyeop Lee** (University of Basel) On the convergence of nonlinear averaging dynamics with three-body interactions on hypergraphs

August 7, 2024

8:00-9:00	**Yoshiko Ogata** (University of Kyoto) Operator algebraic approach to topological orders (I)
9:00-9:30	**Coffee Break**
9:30-10:30	**Yoshiko Ogata** (University of Kyoto) Operator algebraic approach to topological orders (II)
10:30-11:00	**Coffee Break**
11:00-12:00	**Discussion**
12:00-14:00	**Lunch Break**
14:00-15:00	**Mitia Duerinckx** (Université Libre de Bruxelles) New methods for mean-field limits of classical particle systems (III)
15:00-15:30	**Coffee Break**
15:30-15:45	**Florian Haberberger** (LMU Munich) The free energy of dilute Bose gases at low temperatures
15:45-16:00	**Ben Li** (University of Warsaw) Beliaev damping through Friedrichs model
16:00-16:15	**Nguyễn Tòng Xuân** (NYU Shanghai) Local Central Limit Theorem for long-range potentials with continuous spins
16:15-16:30	**Nguyễn Thị Nguyên Khoa** (ENS Paris-Saclay) Development and assessment of physics-informed deep learning methods: towards multiphysics simulation in industrial contexts

August 8, 2024

8:00-9:00	**Mitia Duerinckx** (Université Libre de Bruxelles) New methods for mean-field limits of classical particle systems (IV)
9:00-15:30	**Free time/Field trip**
15:30-17:00	**Robert Seiringer** (IST Austria) Mathematics as the universal language of nature (Public Lecture)

August 9, 2024

9:30-10:30	**Yoshiko Ogata** (University of Kyoto) Operator algebraic approach to topological orders (III)
10:30-11:00	**Coffee Break**
11:00-12:00	**Yoshiko Ogata** (University of Kyoto) Operator algebraic approach to topological orders (IV)
12:00-14:00	**Lunch Break**
14:00-15:00	**Lê Văn Phú Cường** (Heidelberg University) Energy minimising maps and branched optimal transport
15:00-15:30	**Coffee Break**
15:30-16:30	Discussion

August 10, 2024

9:30-10:30	**Robert Seiringer** (IST Austria) Mathematical Foundations of Density Functional Theory (III)
10:30-11:00	**Coffee Break**
11:00-12:00	**Robert Seiringer** (IST Austria) Mathematical Foundations of Density Functional Theory (IV)
12:00-14:00	**Lunch Break**
14:00-15:00	**Marcin Napiórkowski** (University of Warsaw) Free energy asymptotics of the quantum Heisenberg model
15:00-15:30	**Coffee Break**
15:30-16:30	Open problem session

Contents of the lectures

Robert Seiringer: Mathematical Foundations of Density Functional Theory

Abstract: Density Functional Theory (DFT) attempts to describe all the relevant information about the ground state of a many-body quantum system in terms of its one-body reduced density. It is widely and successfully used in practice for computations in quantum chemistry. In these lectures, we shall explain the mathematical structure of DFT, and review known bounds on the universal density functional (including the Lieb-Thirring Inequality for the kinetic energy, and the Lieb-Oxford Inequality for the electrostatic energy). We shall also outline recent results concerning the validity of the local density approximation, and the equivalence of various possible formulations of the uniform electron gas. Much (but not all) of the material of the lectures is contained in the review paper https://arxiv.org/pdf/1912.10424

Mitia Duerinckx: New methods for mean-field limits of classical particle systems

Abstract: One major open problem in mathematical physics still concerns the justification of the Vlasov-Poisson equation as the mean-field limit for systems of classical particles with Coulomb interactions. In this course, we shall review this classical problem, describe some previous approaches, and present a new dual hierarchical method that we recently developed with D. Bresch and P.-E. Jabin. If time allows, we might also discuss related questions regarding correlation estimates and corrections to mean field.

Yoshiko Ogata: **Operator algebraic approach to topological orders**

Abstract: Recently, topological phases of matters have attracted a lot of attention. One characteristic feature of topological phases is the existence of anyons. In this talk, I explain the operator algebraic approach to anyons.

To Tat Dat: The Phi43 measure on compact Riemannian 3-manifolds

Abstract: In this talk, I present a joint work with Dang, Bailleul and Ferdinand in which we construct the Phi43 quantum field theory measure on a compact Riemannian 3-manifold, as an invariant measure of a stochastic partial differential equation. This gives an example of nonperturbative, interacting, non topological quantum field theory constructed on 3-manifolds.

Maxime Van De Moortel: Strong Cosmic Censorship Conjecture and black hole dynamics

Abstract: In the wake of major breakthroughs in General Relativity during the 1960s, Roger Penrose introduced Strong Cosmic Censorship, a profound conjecture regarding the deterministic nature of the theory. Nearly six decades later, Penrose's conjecture remains one of the most captivating unsolved problems in mathematical physics, particularly within the realm of general relativity. Perhaps the most critical arena for testing the conjecture is the spacetime region hidden behind the event horizon: the interior of black holes. The key challenge is to prove the dynamical formation of singularities for solutions to the Einstein equations in this context. We will discuss
recent results on Strong Cosmic Censorship leveraging modern techniques in the theory of quasilinear waves equations at the crossroad of PDEs and Lorentzian geometry.

Jia Shi: Non-radial implosion for compressible Euler and Navier-Stokes in T3 and R3

Abstract: We will discuss the smooth, non-radial solutions of the compressible Euler and Navier-Stokes equation that develop an imploding finite time singularity. The construction is motivated by the radial imploding solutions from Merle--Raphaël--Rodnianski--Szeftel, and Cao-Labora--Buckmaster--Gomez-Serrano but is flexible enough to handle both periodic and non-radial initial data. This is a joint work with Gonzalo Cao-Labora, Javier Gomez-Serrano, and Gigliola Staffilani.

Le Van Phu Cuong: Energy minimising maps and branched optimal transport

Abstract: In their seminal work, Brezis, Coron, and Lieb showed an equivalence between sphere-valued minimising harmonic maps having prescribed topological singularities at given points in R3 and minimal connections between those points, which are solutions of Monge-Kantorovich having those points as marginals. This work was later recast and generalised by Almgren, Browder, and Lieb, who interpreted the minimal connection problem as a suitable Plateau's problem for integral rectifiable currents, having those points as the prescribed boundary. In this talk, we will investigate more general energies for maps with values into Cartesian products of spheres, and investigate their connections to Plateau's problem for currents (or flat chains) with coefficients in suitable groups. In particular, we establish the equivalence for energy minimising configurations of those maps with irrigation Gilbert-Steiner problems. This talk is based on joint work with S. Baldo (Verona), A. Massaccesi (Padova), and G. Orlandi (Verona).

Marcin Napiórkowski: Free energy asymptotics of the quantum Heisenberg model

Abstract: The quantum Heisenberg model, developed by Werner Heisenberg, is a statistical mechanical model used in the study of critical points and phase transitions of magnetic systems, in which the spins of the magnetic systems are treated quantum mechanically. Based on an effective

theory called spin wave approximation, it is believed that low temperature properties of the model can be described in terms of noninteracting quasiparticles called magnons. In my talk I will review the basic concepts and predictions of spin wave approximation and report on recent rigorous results in that direction.

Chanjin You: Phase mixing estimates for the nonlinear Hartree equation of infinite rank

Abstract: The nonlinear Hartree equation for density operators with infinite rank describes the dynamics of infinitely many interacting quantum particles. In this talk, I will present the optimal decay estimates for the density and its derivatives near certain translation-invariant equilibria. The proof relies on establishing pointwise decay estimates for the Green functions in Fourier space and a nonlinear iterative scheme.

Vũ Hồ Thảo Thuận: Hamiltonian approach to 2-layer dispersive stratified fluids

Abstract: A Hamiltonian reduction approach is defined, studied, and finally used to derive asymptotic models of internal wave propagation in density stratified fluids in two-dimensional domains. Beginning with the general Hamiltonian formalism of Benjamin (1986 J. Fluid Mech) for an ideal, stably stratified Euler fluid, the corresponding structure is systematically reduced to the setup of two homogeneous fluids under gravity, separated by an interface and confined between two infinite horizontal plates.

Võ Hoàng Hưng: Liouville Theorems for infinity Laplacian with gradient and KPP type equation

Abstract: In this talk, we prove new Liouville type results for a nonlinear equation involving infinity Laplacian with gradient, of the form $\Delta_\infty^\gamma u + q(x)\cdot\nabla u|\nabla u|^{2-\gamma} +f(x, u)=0,\quad x\in \mathbb{R}^d$ where $\gamma\in [0, 2]$ and $\Delta_\infty^\gamma u$ is a $(3-\gamma)$-homogeneous operator associated with the infinity Laplacian. By assuming $\liminf_{|x|\to\infty}\lim_{s\to0}f(x,s)/s^{3-\gamma}>0$ and q is a continuous function vanishing at infinity, we construct a positive bounded solution to the equation and if $f(x,s)/s^{3-\gamma}$ decreasing in s, we further obtain the uniqueness by improving sliding method for infinity Laplacian operator with nonlinear gradient. Otherwise, if $\limsup_{|x|\to\infty}\sup_{[\delta_1,\delta_2]}f(x,s)<0$, then nonexistence result holds provided additionally some suitable conditions. To this aim, we develop novel techniques to overcome the difficulties stemming from the degeneracy of infinity Laplacian and nonlinearity of the gradient term. Our approach is based on a new regularity result, the strong maximum principle, and Hopf's lemma for infinity Laplacian involving gradient and potential. We also construct some examples to illustrate our results. We further investigate some deeper qualitative properties of the principal eigenvalue of the corresponding nonlinear operator $\Delta_\infty^\gamma u + q(x)\cdot \nabla{u} |\nabla{u}|^{2-\gamma} + c(x)u^{3-\gamma},$ with Dirichlet boundary condition in smooth bounded domains, which may be of independent interest. The results obtained here could be considered as sharp extension of the Liouville type results obtained previously.

Nguyễn Tiến Tài: Nonlinear Rayleigh-Taylor instability in fluid mechanics and beyond

Abstract: The study of the stability of laminar flows satisfying a system of hyperbolic equations has attracted a lot of attention of physicians and mathematicians due to its appearance in numerous models in fluid mechanics, e.g. Rayleigh-Taylor, Kelvin- Hemholtz, Zeldovitch-von Neumann-Doring detonation. In this talk, I consider a particular model, the viscous Rayleigh-Taylor instability for a smooth increasing density profile and discuss my recent results in some settings, by following the framework of Guo-Strauss '95 and Grenier '00 with a refinement. Some perspectives will be introduced if time allows.

Jinyeop Lee: On the convergence of nonlinear averaging dynamics with three-body interactions on hypergraphs

Abstract: In many complex networked systems, interactions happens not only with simple pairs but also through multi-body interactions. We explore a discrete-time dynamics with three-body interactions on a 3-uniform hypergraph, where nodes update their states through a nonlinearly-weighted average of neighboring pairs' states. This captures reinforcing group effects. Unlike linear averaging dynamics, our model does not converge to the initial states' average but to a some shifted value.Under random initial states and certain hypergraph conditions, we prove convergence to a multiplicatively-shifted average with high probability.

Florian Haberberger: The free energy of dilute Bose gases at low temperatures

Abstract: I will discuss dilute Bose gases in a many-body quantum mechanical model. A formula for the free energy, i.e. the minimal energy of the system at positive temperature is presented. I will then give the key idea for the proof of the upper bound, which is the most recent result. This is joint work with Hainzl,C.; Nam, P.T.; Schlein,B.; Seiringer, R. and Triay, A.

Ben Li: Beliaev damping through Friedrichs model

Abstract: We consider the Bose gas at zero temperature. According to Bogoliubov theory the low energy behaviour of the system is described by non-interacting bosonic quasiparticles called phonons. In this presentation we show how the mechanism of Beliaev damping - the decay of phonons due to interactions between them - can be explained using an effective theory based on the Friedrichs model approach. This allows us to formulate in a rigorous way a theorem that describes this phenomenon. In particular, we compute the damping ratio of phonons - a result first derived by S.T. Beliaev using diagrammatic techniques.

Nguyễn Tòng Xuân: Local Central Limit Theorem for long-range potentials with continuous spins

Abstract: We prove the integral central limit theorem implies the local central limit theorem for potentials with long-range interactions on the lattice Zd where the state space is compact and also the set of real numbers. The proof works for any temperature. Our method relies on the control of certain characteristic functions on three different regions. This talk is based on the joint work with R. Fernandez, Eric O. Endo, and Vlad Margarint.

Nguyễn Thị Nguyên Khoa: Development and assessment of physics-informed deep learning methods: towards multiphysics simulation in industrial contexts

Abstract: Physics-Informed Neural Networks (PINNs) have gained much attention in various engineering fields thanks to their capability of incorporating physical laws, which are often represented by Partial Differential Equations (PDEs), into the models. However, assessing PINNs in complex multiphysics problems relevant to industrial contexts is still an active research topic as the vanilla PINNs often fail in these practical scenarios. The first part of this work focuses on improving PINNs accuracy by using adaptive strategies on the training points. Different from other existing approaches, our proposed method aims to capture not only the global extrema but also the local extrema of the PDEs residuals, thus improving the accuracy and reducing the training time. In the second part, we investigate the geometry-aware frameworks for PINNs and propose a novel version for the deep energy PINNs-based method. This approach leverages the weak form of the physical system equation and minimizes the loss function based on the potential energy of all considered geometries. It is expected that these geometry-aware frameworks can infer the solution on various shapes of geometry using only one trained model. All the proposed approaches in this work undergo different industrial test cases at Michelin and CEA and demonstrate their efficiency when dealing with these complex problems, including the rubber calendering process and tire loading simulation in tire manufacturing, and the downscaling of geographic gravity waves in the climate modeling.