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PAN ASIA NUMBER THEORY CONFERENCE 2024

Time: July 15-19, 2024

Venue: Hanoi, Vietnam

This is the booklet for the Pan Asia Number Theory Conference 2024.
Further information can be found at:
`https://viasm.edu.vn/en/hdkh/PANT2024`

Contents

About

Pan Asia Number Theory Conference 2024

The PANT conference was initiated by Professor John Coates and a group of established number theorists who were either based in Asia or of Asian origin. The goal of the conference is to showcase exciting recent developments in number theory and related subjects, especially those occurring in the greater Asian regions or featuring works by mathematicians of Asian descent. It also aims to provide a platform for Asian number theorists to interact and collaborate with each other, especially those of the younger generation. PANT had been an annual week-long conference whose venue rotated among several countries.

Previous PANT Conferences have been held in Korea (2009, 2014), Japan (2010, 2021), China (2011, 2015, 2023), India (2012), Vietnam (2013), Taiwan (2016), Singapore (2018).

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Akshay Venkatesh (Stanford University and Institute for Advanced Study, USA)

Invited Speakers

Hung Manh Bui (University of Manchester, UK)
Ashay Burungale (The University of Texas at Austin, USA)
Chien-Hua Chen (National Taiwan University, Taiwan)
Jingren Chi (Chinese Academy of Sciences, China)
Sungyoon Cho (Pohang University of Science and Technology, Korea)
Shaunak Deo (Indian Institute of Science, India)
Yujiao Jiang (Shandong University, China)
Mahesh Kakde (Indian Institute of Science, India)
Hiroki Kato (Max Planck Institute for Mathematics, Germany)
Chan-Ho Kim (Ewha Womans University, Korea)
Bao Viet Le Hung (Northwestern University, USA)
Zhilin Luo (University of Chicago, USA)
Kentaro Nakamura (Saga University, Japan)
Tuan Ngo Dac (CNRS - Université de Caen Normandie, France)
Kieu Hieu Nguyen (Institut de Mathématiques de Marseille, France)
Masao Oi (Kyoto University, Japan)
Emile Takahiro Okada (National University of Singapore, Singapore)
Arghya Sadhukhan (National University of Singapore, Singapore)
Daichi Takeuchi (RIKEN Center for Advanced Intelligence Project, Japan)
Haowu Wang (Wuhan University, China)
Hongbo Yin (Shandong University, China)
Alex Youcis (National University of Singapore, Singapore)
Yihang Zhu (Tsinghua University, China)

Sponsors

The Pan Asia Number Theory Conference 2024 is organised by the Vietnam Institute for Advanced Study in Mathematics (VIASM), in partnership with the Clay Mathematics Institute (CMI). The Organisers gratefully acknowledge the support of:

- Vietnam Institute for Advanced Study in Mathematics (VIASM),

- Vietnam National Program for the Development of Mathematics 2021-2030 (NPDM),

- Clay Mathematics Institute (CMI),

- Number Theory Foundation (NTF),

- Alabaster.

Timetable

All the talks are scheduled in Hanoi time (GMT+7).

Monday, July 15

8:00–9:00	Registration	
9:00–9:10	Welcome remarks	
9:10–10:10	**Tuan Ngo Dac** CNRS - Université de Caen Normandie, France	*On alternating multiple zeta values in positive characteristic*
10:10–10:30	Teatime	
10:30–11:30	**Chien-Hua Chen** National Taiwan University, Taiwan	*On singular moduli for higher rank Drinfeld modules*
11:30–13:30	Lunch	
13:30–14:30	**Alex Youcis (remote)** National University of Singapore, Singapore	*Serre-Tate theory for Shimura varieties of abelian type*
14:30–14:45	Teatime	
14:45–15:45	**Sungyoon Cho** Pohang University of Science and Technology, Korea	*On the Kudla-Rapoport conjecture at a place of bad reduction*
15:45–16:00	Teatime	
16:00–17:00	**Hongbo Yin** Shandong University, China	*The sum of two rational cubes*

Tuesday, July 16

9:00–10:00	**Yihang Zhu** Tsinghua University, China	*Zeta Functions of Shimura Varieties: Past, Present, and the Near Future*
10:00–10:15	Teatime	
10:15–11:15	**Jingren Chi** Chinese Academy of Sciences, China	*Langlands-Kottwitz method for the bad reduction of simple Shimura varieties*
11:15–11:30	Teatime	
11:30–12:30	**Ashay Burungale** The University of Texas at Austin, USA	*Hecke L-values, definite Shimura sets and mod p non-vanishing*
12:30–14:00	Lunch	
14:00–15:00	**Hung Manh Bui** University of Manchester, UK	*Negative moments of the Riemann zeta-function*
15:00–15:20	Teatime	
15:20–16:20	**Yujiao Jiang** Shandong University, China	*Correlations of multiplicative functions*
17:00–18:30	Welcome reception	

Wednesday, July 17

9:00–10:00	**Kieu Hieu Nguyen** Institut de Mathématiques de Marseille, France	*On categorical local Langlands for GL_n*
10:00–10:15	Teatime	
10:15–11:15	**Bao Viet Le Hung** Northwestern University, USA	*The generic Fontaine-Laffaille spectral Hecke algebra*
11:15–11:30	Teatime	
11:30–12:30	**Chan-Ho Kim** Ewha Womans University, Korea	*On the adjoint Selmer groups of semi-stable elliptic curves and Flach's zeta elements*
12:30–14:00	Lunch	

Thursday, July 18

9:00–10:00	**Daichi Takeuchi** RIKEN Center for Advanced Intelligence Project, Japan	*Deligne-Laumon formula for local epsilon factor*
10:00–10:15	Teatime	
10:15–11:15	**Kentaro Nakamura** Saga University, Japan	*Local epsilon conjecture and p-adic differential equations*
11:15–11:30	Teatime	
11:30–12:30	**Masao Oi** Kyoto University, Japan	*On a characterization of Deligne-Lusztig representations via their characters*
12:30–14:00	Lunch	
14:00–15:00	**Arghya Sadhukhan** National University of Singapore, Singapore	*Dimension formula for (closed) affine Deligne-Lusztig varieties of parahoric level*
15:00–15:20	Teatime	
15:20–16:20	**Hiroki Kato** Max-Planck Institute for Mathematics, Germany	*ℓ-independence via motives*

Friday, July 19

9:00–10:00	**Mahesh Kakde** Indian Institute of Science, India	*Refinements of the Brumer–Stark conjecture and ETNC*
10:00–10:15	Teatime	
10:15–11:15	**Shaunak Deo** Indian Institute of Science, India	*Level raising for modular forms in the residually reducible case*
11:15–11:30	Teatime	
11:30–12:30	**Haowu Wang** Wuhan University, China	*Hyperbolization of affine Lie algebras*
12:30–14:00	Lunch	
14:00–15:00	**Emile Takahiro Okada** National University of Singapore, Singapore	*Harmonic analytic and spectral characterisations of Arthur packets*
15:00–15:20	Teatime	
15:20–16:20	**Zhilin Luo** University of Chicago, USA	*Nonabelian Fourier kernels for SL_2 and GL_2*

Talk Abstracts

Monday, July 15

On alternating multiple zeta values in positive characteristic

Tuan Ngo Dac   **CNRS - Université de Caen Normandie, France**

Zagier-Hoffman's conjectures predict the dimension and a basis for the $\mathbb{Q}$-vector spaces spanned by Nth cyclotomic multiple zeta values (MZV's) of fixed weight, where N is a natural number. For $N = 1$ (MZV's case), half of these conjectures have been solved by the work of Terasoma, Deligne-Goncharov and Brown using Zagier's identity. The other half is completely open. For $N = 2$ (alternating MZV's case), Deligne-Goncharov and Deligne solved the same half of these conjectures for alternating MZV's. We will briefly review these works. Then, by analogy we focus on the analog of the Zagier-Hoffman conjectures for alternating MZV's in positive characteristic and report our work in this direction. This is a joint work with Im Bo-Hae, Kim Hojin, Le Khac Nhuan, and Pham Lan Huong.

On singular moduli for higher rank Drinfeld modules

Chien-Hua Chen   **National Taiwan University, Taiwan**

As a function field analogue of singular moduli for elliptic curves estimated by Gross-Zagier, we estimate the valuation at certain places of singular moduli for prime rank Drinfeld modules. Our estimation can be viewed as a generalization of the rank-2 case proved by Dorman. This talk consists of three parts:

Firstly, we compare the valuation of singular moduli with the number of isomorphisms between "Drinfeld module with CM by the ring of integer of an imaginary extension over $\mathbb{F}_q(T)$" and "a specific prime rank Drinfeld module with CM by a constant extension of the polynomial ring $\mathbb{F}_q(T)$". Due to the difference between the structure of moduli schemes for Drinfeld modules of rank and that for Drinfeld modules of rank 2 (which is similar to the elliptic curve case), this comparison can only result in an inequality relation.

Secondly, we reduce counting the number of isomorphisms into counting the number of certain endomorphisms on a Drinfeld module whose reduced characteristic polynomial is of certain form. This reduction step makes the counting process more concrete and computable.

Lastly, we compute some examples on singular moduli estimation for rank 3 Drinfeld modules.

Serre-Tate theory for Shimura varieties of abelian type

Alex Youcis   **National University of Singapore, Singapore**

The celebrated Serre–Tate theorem says that deformations of an abelian variety are naturally parameterized in terms of deformation of the abelian variety's Barsotti–Tate group. In particular, this says that the natural functor from Mumford's moduli spaces of principally polarized abelian varieties to the moduli stack of Barsotti–Tate groups is formally étale. In this talk I will discuss joint work with Naoki Imai and Hiroki Kato which shows a similar result holds true for arbitrary Shimura varieties of abelian type (at hyperspecial level), for which Mumford's moduli spaces are very specific examples of.

On the Kudla-Rapoport conjecture at a place of bad reduction

Sungyoon Cho   **Pohang University of Science and Technology, Korea**

The Kudla-Rapoport conjecture predicts a relation between the arithmetic intersection numbers of special cycles on a unitary Shimura variety and the derivative of representation densities for hermitian forms at a place of good reduction. In this talk, I will present a variant of the Kudla-Rapoport conjecture at a place of bad reduction. Additionally, I will discuss a proof of the conjecture in several new cases in any dimension. This talk is partially based on joint work with Qiao He and Zhiyu Zhang.

The sum of two rational cubes

Hongbo Yin   **Shandong University, China**

One old question in number theory is to determine whether an integer can be written as the sum of two nonzero rational cubes. The Sylvester conjecture predicts that for every prime p congruent to $4, 7, 8$ modulo 9, the answer is positive. This conjecture is open and only has some partial results. In this talk, I will introduce the background of this problem and some progress in recent years.

Tuesday, July 16

Zeta Functions of Shimura Varieties: Past, Present, and the Near Future

Yihang Zhu   **Tsinghua University, China**

I will first recall the general expectations of Shimura, Langlands, and Kottwtiz on the shape of the zeta function of a Shimura variety, or more generally its étale cohomology. I will then report on some recent progress which partially fulfills these expectations, for Shimura varieties of unitary groups and special orthogonal groups. Finally, I will give a preview of some foreseeable developments in the near future.

Langlands-Kottwitz method for the bad reduction of simple Shimura varieties

Jingren Chi   **Chinese Academy of Sciences, China**

In this talk we first introduce the attempts of Rapoport, Scholze and Scholze-Shin to extend the Langlands-Kottwitz method of describing étale cohomology of Shimura varieties to cases of bad reduction (in which the level structure could be arbitrary and the local group could be non-quasi-split). In their approach, the problem is reduced to certain conjectures on orbital integrals and characters of local test functions that are of independent interest. Then I will explain some progress on these conjectures for inner forms of $\mathrm{GL}(n)$. This is based on joint work with Thomas Haines.

Hecke L-values, definite Shimura sets and mod p non-vanishing

Ashay Burungale   **The University of Texas at Austin, USA**

We outline mod p non-vanishing of Hecke L-values in self-dual families over imaginary quadratic fields. This includes the vanishing of the mu-invariant of Rubin's p-adic L-function. (Joint with W. He, S. Kobayashi and K. Ota.)

Negative moments of the Riemann zeta-function

Hung Manh Bui   **University of Manchester, UK**

Conditional on the Riemann Hypothesis, Soundararajan proved upper bounds of almost the right order of magnitude for positive moments of the Riemann zeta-function. This was later strengthened by Harper. We will discuss recent joint work with Alexandra Florea where we consider negative moments of the Riemann zeta-function. We will explain how we can obtain asymptotic formulas for negative moments when the shift in the zeta-function is large enough, confirming a conjecture of Gonek, and also discuss how to obtain non-trivial upper bounds when the shifts are smaller.

Correlations of multiplicative functions

Yujiao Jiang   **Shandong University, China**

Understanding the correlations of multiplicative functions is a central question in analytic number theory and is related to various celebrated unsolved problems, including Chowla's conjecture on the autocorrelation of Mobius function and the additive divisor problem. In this talk, we shall present our work concerning the correlation of two multiplicative functions. As examples, we make some breakthroughs on shifted convolution problems for $\mathrm{GL}(n){\times}\mathrm{GL}(2)$ $(n \geq 4)$ and Hypothesis C of Iwaniec-Luo-Sarnak for the first time. This is joint work with Guangshi Lü.

Wednesday, July 17

On categorical local Langlands for GL_n

Kieu Hieu Nguyen   **Institut de Mathématiques de Marseille, France**

Recently, Fargues-Scholze and many other people realized that there should be a categorical version which encodes great information of the local Langlands correspondence. In this talk, I will describe the objects appearing in their conjectures and explain some relations with the local Langlands correspondences for GL_n.

The generic Fontaine-Laffaille spectral Hecke algebra

Bao Viet Le Hung   **Northwestern University, USA**

The mod p categorical Langlands program conjectures an embedding of the (derived) category of mod p smooth representations of $\mathrm{GL}_n(\mathbb{Q}_p)$ into a suitable (derived) category of quasi-coherent sheaves on the Emerton-Gee stack X classifying n-dimensional representations of the absolute Galois group of $\mathbb{Q}_p$. In particular, this predicts an identification of the derived endomorphism of certain standard representations of $\mathrm{GL}_n(\mathbb{Q}_p)$ and the derived endomorphism of specific sheaves on X. I will explain the computation of the cohomology algebra of the latter in the most accessible case, where the sheaf is expected to be a line bundle on a well-understood locus (the Fontaine-Laffaille locus). This is joint work in progress with Junho Won.

On the adjoint Selmer groups of semi-stable elliptic curves and Flach's zeta elements

Chan-Ho Kim   **Ewha Womans University, Korea**

We explicitly construct the rank one primitive Stark (equivalently, Kolyvagin) system extending a constant multiple of Flach's zeta elements for semi-stable elliptic curves. As its arithmetic applications, we obtain the equivalence between a specific behavior of the Stark system and the minimal modularity lifting theorem, and we also discuss the cyclicity of the adjoint Selmer groups. Our Stark system construction yields a more refined interpretation of the collection of Flach's zeta elements than the "geometric Euler system" approach due to Flach, Wiles, Mazur, and Weston.

Thursday, July 18

Deligne-Laumon formula for local epsilon factor

Daichi Takeuchi   **RIKEN Center for Advanced Intelligence Project, Japan**

Let X be a smooth curve over a field and U be the open complement of a closed point x in X. For a locally constant étale sheaf on U, its Swan conductor is an important invariant to understand the ramification of the sheaf around the point x. Around 1980, Deligne considered the problem of how the Swan conductor varies if the curve and the sheaf vary in a family, and he proved that the gap of the conductors is controlled by the rank of the vanishing cycles.

In this talk, I consider a similar question for local epsilon factor, instead of Swan conductor and prove that the gap of the local epsilon factors is controlled by the determinant of the vanishing cycles.

Local epsilon conjecture and p-adic differential equations

Kentaro Nakamura   **Saga University, Japan**

Local epsilon conjecture is a conjecture on the interpolation of the de Rham trivializations to all the families of p-adic Galois representations. The de Rham trivializations are explicit trivializations of the determinants of Galois cohomologies of de Rham representations in terms of p-adic Hodge theoretic datum, such as local epsilon- and L-factors and Bloch-Kato exponentials. In this talk, we focus on this conjecture for the cyclotomic deformations of de Rham representations, more generally, de Rham (phi,Gamma)-modules over the Robba ring. We show that the conjecture for the cyclotomic deformation of any de Rham (phi,Gamma)-module is equivalent to that for the associated p-adic differential equation defined by Laurent Berger. This is a joint work with Tetsuya Ishida.

On a characterization of Deligne-Lusztig representations via their characters

Masao Oi   **Kyoto University, Japan**

A fundamental theorem in representation theory of finite groups says that any representation of a finite group is uniquely determined by its characters, i.e., traces on all conjugacy classes. However, sometimes it is possible to determine a given representation only by looking at its characters on some specific conjugacy classes. For application, it is convenient if we can find such a set such that the characters there have a "simple form". In this talk, I discuss this problem for Deligne-Lusztig representations, which are representations of finite reductive groups. I will also explain its relation/application to representation theory of p-adic groups and the local Langlands correspondence. This is a joint work with Charlotte Chan (Univ. of Michigan).

Dimension formula for (closed) affine Deligne-Lusztig varieties of parahoric level

Arghya Sadhukhan   **National University of Singapore, Singapore**

Modeled after their classical counterparts in the context of affine root systems, the affine Deligne-Lusztig variety $X^G(\mu, b)_K$ is closely related to Rapoport-Zink spaces and moduli spaces of G-shtukas. Their geometric structure is in general very complicated, and many questions about them are primarily resolved at the hyperspecial level. In this talk, I will discuss my recent work toward a dimension formula for $X^G(\mu, b)_K$ for arbitrary parahoric levels in a quasi-split group, as well as some partial descriptions of the dimension and top-dimensional irreducible components in the non-quasi-split case; in particular, we will identify some naturally occuring iterated fibrations over classical Deligne-Lusztig varieties amongst these components.

ℓ-independence via motives

Hiroki Kato   **Max-Planck Institute for Mathematics, Germany**

For an algebraic variety or a rigid space (with mild assumptions) over a p-adic field, its ℓ-adic étale cohomology groups are naturally equipped with a Galois action (in some appropriate sense in the $\ell = p$ case). I am going to discuss ℓ-independence of the alternating sum of traces for this action. I propose a uniform approach using recent developments in the theory of rigid analytic motives. Especially, the result (in progress) includes some new cases, such as the case of certain non-quasi-compact rigid spaces (ones associated with formal schemes formally of finite type) and the case of p-adic cohomology (overconvergent Hyodo–Kato cohomology) of such rigid spaces.

Friday, July 19

Refinements of the Brumer–Stark conjecture and ETNC

Mahesh Kakde   **Indian Institute of Science, India**

Equivariant Tamagawa Number Conjecture (ETNC) relates leading term of L-function of a motive to arithmetic invariants of the motive. It seems to encompass most known theorems and conjectures about special values and leading terms of L-functions of motives. Even though we have this nice conjectural picture, very few cases of this conjecture are completely known. For instance, it is unknown for one dimensional characters of a number field. One case, in which we now know a complete answer is the case of totally odd characters of totally real number fields. An important ingredient is a strong version of the Brumer-Stark conjecture. In the first half of the talk I will review the Brumer–Stark conjecture and its refinements. In the second half I will explain the statement of Equivariant Tamagawa Number Conjecture for totally odd characters of totally real number fields and sketch a proof. This proof is largely inspired by the proof of ETNC given by Bullach–Burns–Daoud–Seo who, more generally, developed the theory of "scarcity of Euler systems". This is joint work with Samit Dasgupta, Jesse Silliman and Jiuya Wang.

Level raising for modular forms in the residually reducible case

Shaunak Deo   **Indian Institute of Science, India**

Suppose p is a prime, F is a finite field of characteristic p and $\bar{\rho} : G_{\mathbb{Q}} \to \mathrm{GL}_2(F)$ is a continuous, odd, semi-simple, reducible representation. In this setting, one can ask the following natural question: What are all the levels at which there exists a newform lifting $\bar{\rho}$? We will discuss some results in this direction. These results can be thought of as simultaneous level raising at multiple primes (in the spirit of level raising results of Diamond-Taylor in the irreducible case). Deformation theory of Galois (pseudo-)representations plays a key role in the proofs of these results which we hope to highlight in the talk.

Hyperbolization of affine Lie algebras

Haowu Wang **Wuhan University, China**

In 1983, Feingold and Frenkel posed a question about possible relations between affine Lie algebras, hyperbolic Kac–Moody algebras and Siegel modular forms. We give an automorphic answer to this question and its generalization. We classify Borcherds–Kac–Moody algebras whose denominators define reflective automorphic products of singular weight. As a consequence, we prove that there are exactly 81 affine Lie algebras which have nice extensions to BKM algebras. We find that 69 of them appear in Schellekens' list of holomorphic CFT of central charge 24, while 8 of them correspond to the $N = 1$ structures of holomorphic SCFT of central charge 12 composed of 24 chiral fermions. The last 4 cases are related to exceptional modular invariants from nontrivial automorphisms of fusion algebras. This clarifies the relationship of affine Lie algebras, vertex algebras and hyperbolic BKM superalgebras at the level of modular forms. This is based on a joint paper with Kaiwen Sun and Brandon Williams.

Harmonic analytic and spectral characterisations of Arthur packets

Emile Takahiro Okada **National University of Singapore, Singapore**

In their study of the unitary dual for complex reductive groups, Barbasch and Vogan gave a characterisation of the unipotent Arthur packets using a microlocal invariant called the wavefront set. In this talk I will present joint results with Maxim Gurevich in which we pursue this approach for classical p-adic groups. We find that the packets defined microlocally are a union of Arthur packets which can be characterised by a surprising spectral property, namely the existence of an invariant vector for a (not necessarily hyperspecial) maximal compact subgroup.

Nonabelian Fourier kernels for SL_2 and GL_2

Zhilin Luo **University of Chicago, USA**

I will present joint work with B.C. Ngô unveiling an explicit formula for nonabelian Fourier kernels in $G = \mathrm{SL}_2$ and GL_2 that is conjectured by A. Braverman and D. Kazhdan. I will also explain the connection with the orbital Hankel transform suggested by Ngô.

Useful Information

The **Pan Asia Number Theory Conference 2024 (PANT2024)** (July 15 - July 19) will be after the **VIASM Annual Meeting 2024** (July 13), also held at the **Vietnam Institute for Advanced Study in Mathematics (VIASM)**.

Talks will be held at the **Laurent Schwartz Lecture Hall** of the VIASM. **Teatime** will be at the hallway outside of the Lecture Hall. **Wi-Fi** will be available; the detailed information of Wi-Fi access will be provided during the conference.

How to get to VIASM?

The address of the Vietnam Institute for Advanced Study in Mathematics (VIASM) is:

157 Chùa Láng Street, Láng Thượng Ward, Đống Đa District, Hà Nội, Việt Nam.

In English: 157 Chua Lang Street, Lang Thuong Ward, Dong Da District, Hanoi, Vietnam.

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