## International Conference on Commutative Algebra to the Memory of Jürgen Herzog

Vietnam Institute for Advanced Study in Mathematics, VIASM 21-25 July 2025

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Location: Vietnam Institute for Advanced Study in Mathematics 161 Huynh Thuc Khang Street, Lang Ha Ward, HN

**Venue**: Laurent Schwartz Lecture Hall, 1st floor, Vietnam Institute for Advanced Study in Mathematics

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- Aldo Conca, Università di Genova, Italy
- Takayuki Hibi, Osaka University, Japan
- Tim Römer, Universität Osnabrück, Germany
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## I. PROGRAM

## Monday, July 21, 2025

14:45 – 15:00 **Coffee break** 

Morning

| 0                  |   |  |
|--------------------|---|--|
| Chair: Le Tuan Hoa |   |  |
| 07:30 - 08:45      | Registration  |  |
| 08:45 - 09:00      | Opening   |  |
| 09:00 - 09:45      | <b>Bernd Ulrich</b> (Purdue University)<br>The syzygies of the residue field of Golod rings         |  |
| 09:45 - 10:00      | Coffee break  |  |
| 10:00 - 10:45      | Hai Long Dao (University of Kansas)<br>On componentwise linear ideals                               |  |
| 11:00 - 11:45      | <b>Hidefumi Ohsugi</b> (Kwansei Gakuin University)<br>Commutative algebra and graph coloring theory |  |
| A fternoon         |   |  |
| Chair: Aldo C      | Conca   |  |
| 14:00 - 14:45      | Winfried Bruns (Universität Osnabrück)  |  |

Jürgen Herzog: life and work

15:00 – 15:45 Naoki Terai (Okayama University)

#### 4

The v-numbers of squarefree monomial ideals

16:00 – 16:45 **Hop D. Nguyen** (Institute of Mathematics, VAST) Koszul property and finite linearity defect over g-stretched local rings

## Tuesday, July 22, 2025

Morning

#### Chair: Hai Long Dao

| 09:00 - 09:45 | <b>David Eisenbud</b> (MSRI, Berkeley) |
|---------------|--|
|               | Observations (and a few results) on    |
|               | infinite free resolutions              |

- 09:45 10:00 Group picture & Coffee break
- 10:00 10:45 **Giulio Caviglia** (Purdue University) Quadratic linear strands of prime ideals
- 10:55 11:10 Arindam Banerjee (IIT Kharagpur) Some results on Castelnuovo–Mumford regularity of ideals related to graphs
- 11:15 11:30 **Guangjun Zhu** (Soochow University) Generalized binomial edge ideals of bipartite graphs
- 11:35 11:50 **Ramakrishna Nanduri** (IIT Kharagpur) On toric ideals of weighted oriented graphs

#### Afternoon

### Chair: Hema Srinivasan

| 14:00 - 14:45 | Claudia Polini (University of Notre Dame)<br>Behrend function and blowup algebras |
|---------------|---|
| 14:45 - 15:00 | Coffee break  |
| 15:00 - 15:45 | <b>Matteo Varbaro</b> (Università di Genova)<br>Singularities of Herzog varieties |
| 16.00 - 16.45 | Emanuela De Negri (Università di Cenova)  |

16:00 – 16:45 **Emanuela De Negri** (Università di Genova) Invariants of toric double determinantal rings

## Wednesday, July 23, 2025

Morning

## Chair: Jugal Verma

| 09:00 - 09:45 | <b>Steven Dale Cutkosky</b> (University of Missouri)<br>Degree functions through intersection theory                              |
|---------------|---|
| 09:45 - 10:00 | Coffee break  |
| 10:00 - 10:45 | <b>Tài Huy Hà</b> (Tulane University)<br>Asymptotic regularity of graded families<br>of homogeneous ideals                        |
| 10:55 - 11:10 | <b>Kazuho Ozeki</b> (Nihon University)<br>The first Hilbert coefficient of stretched ideals                                       |
| 11:15 - 11:30 | <b>Mousumi Mandal</b> (IIT Kharagpur)<br>Upper bounds for second Hilbert coefficients   |
| 11:35 - 11:50 | <b>Dipankar Ghosh</b> (IIT Kharagpur)<br>Complexity and curvature of (pairs of) Cohen–Macaulay<br>modules, and their applications |

## A fternoon

## Chair: Jugal Verma

- 14:00 14:45 **Sara Saeedi Madani** (Amirkabir Univ. of Technology) Binomial edge ideals and some other related ideals
- 14:45 15:00 **Coffee break**
- 15:00 17:00 **Poster Session**

## Thursday, July 24, 2025

Morning

#### Chair: Tim Römer

| 09:00 - 09:45 | Kei-ichi Watanabe (Nihon University)                  |
|---------------|---|
|               | Almost Gorenstein and nearly Gorenstein properties of |
|               | 2-dimensional normal rings; using                     |
|               | resolution of singularities                           |
| 09:45 - 10:00 | Coffee break  |
| 10:00 - 10:45 | Volkmar Welker (Philipps-Universität Marburg)         |

- Generalized binomial edge ideals and the Cartwright-Sturmfels property
- 10:55 11:10 **Clare D'Cruz** (Chennai Mathematical Institute) Joint reduction and adjoint of ideals
- 11:15 11:30 **Pham Hong Nam** (Thai Nguyen Univ. of Sciences) Unmixed torsions and sequentially Cohen–Macaulay modules

### 11:35 – 11:50 **Shreedevi K. Masuti** (IIT Dharwad) Artinian Gorenstein algebras with a binomial Macaulay dual generator

#### Afternoon

#### Chair: Tài Huy Hà

| 14:00 - 14:45 | Marc Chardin (Sorbonne Université)             |
|---------------|--|
|               | Cohomology and free resolutions over a product |
|               | of projective spaces                           |

- 14:45 15:00 **Coffee break**
- 15:00 15:45 **Tony J. Puthenpurakal** (IIT Bombay) On lengths of modules over certain Artinian complete intersections
- 16:00 16:45 **Zhongming Tang** (Soochow University) Symmetric algebras and s-sequences
- 17:30 19:30 **Conference banquet** at Vietnam Institute for Advanced Study in Mathematics

## Friday, July 25, 2025

Morning

#### Chair: Zhongming Tang

| 09:00 - 09:45 | Le Tuan Hoa (Institute of Mathematics, VAST)    |
|---------------|---|
|               | New bounds on Castelnuovo-Mumford regularity of |
|               | monomial curves and application to sumsets      |

- 09:45 10:00 Coffee break
- 10:00 10:45 **Hema Srinivasan** (University of Missouri) Numerical semigroups inspired by Judy Sally
- 11:00 11:45 **Dumitru Stamate** (University of Bucharest) Asymptotic properties for shifted families of numerical semigroups

#### Afternoon

#### Chair: Ngo Viet Trung

- 14:00 14:45 **Srikanth Iyengar** (University of Utah) On Herzog's conjecture on cotangent homology of commutative algebras
- 14:45 15:00 Coffee break
- 15:00 15:45 Ayesha Asloob Qureshi (Sabancı University) On squarefree powers of simplicial trees
- 16:00 16:45 **Jugal Verma** (Indian Inst. of Tech. Gandhinagar) Joint reductions of complete modules and their mixed Buchsbaum–Rim polynomials

# **II. ABSTRACTS OF TALKS**

## 1. Some results on Castelnuovo–Mumford regularity of ideals related to graphs

Arindam Banerjee (Indian Institute of Technology, Kharagpur, 123.arindam@gmail.com)

Castelnuovo–Mumford regularity of ideals with some underlying combinatorial structure has been a very active area of research in last few decades. In this talks we shall discuss few recent results on regularity of edge ideals and weighted oriented edge ideals.

#### 2. Jürgen Herzog: life and work

Winfried Bruns (Universität Osnabrück, wbruns@uos.de)

I will give an overview of Jürgen Herzog's life and his most important mathematical achievements. Some pictures will help us to refresh our memories of a remarkable colleague and friend.

#### 3. Quadratic linear strands of prime ideals

Giulio Caviglia (*Purdue University, gcavigli@purdue.edu*)

We prove sharp estimates on the quadratic strand of the resolution of any homogeneous prime ideal in a standard graded polynomial ring over an arbitrary field. Our bounds only depend on the height of the prime ideal, and they are optimal since for every  $h \ge 1$  we show that there exists a prime ideal of height h achieving them. In particular, we show that a prime ideal of height h can contain at most  $h^2$  quadratic minimal generators, and that there exists a prime ideal minimally generated by  $h^2$ quadrics.

This is joint work with Alessandro De Stefani.

# 4. Cohomology and free resolutions over a product of projective spaces

Marc Chardin (Sorbonne Université, mchardin@imj-prg.fr)

In the standard graded situation (one projective space), the Castelnuovo–Mumford regularity and the  $a^*$ -invariant of a graded module provide two links between cohomology vanishing and shifts in a minimal free resolution. I will report about how this correspondence, and other classical results about Hilbert functions, complete intersections or degree truncations of modules, extend to the case of a standard multigraded polynomial algebra. This is based on the work of many colleagues and collaborators and includes some recent advances.

#### 5. Degree functions through intersection theory

Steven Dale Cutkosky (University of Missouri, CutkoskyS@missouri.edu)

We prove some theorems about intersection theory above a local ring, and obtain as a corollary a geometric proof of a theorem of Rees about degree functions. We discuss an extension of degree functions to graded families of ideals. This is joint work with Jonathan Montaño.

#### 6. On componentwise linear ideals

Hai Long Dao (University of Kansas, hdao@math.ku.edu)

Componentwise linear ideals were introduced by Herzog-Hibi and are now a very active area of research. Let I, J be componentwise linear ideals in a polynomial ring S. We study necessary and sufficient conditions for I + J to be componentwise linear. We provide a complete characterization when dim S = 2. As a consequence, any componentwise linear monomial ideal in k[x, y] has linear quotients using generators in non-decreasing degrees. In any dimension, we show that under mild compatibility conditions, one can build a componentwise linear ideal from a given collection of componentwise linear monomial ideals using only sum and product with square-free monomials. We provide numerous examples to demonstrate the optimality of our results. This is joint work with Sreehari Suresh-Babu.

#### 7. Invariants of toric double determinantal rings

Emanuela De Negri (Università di Genova, emanuela.denegri@unige.it)

Double determinantal rings and varieties were introduced by Li as instances of Nakajima quiver varieties, but they are also a natural generalization of classical determinantal rings.

In this talk, we focus on toric double determinantal rings and show that they coincide with the Hibi rings associated to certain finite distributive lattices. Using this fact we compute the number of minimal generators, the multiplicity, the regularity, the *a*-invariant and the Hilbert function of these toric rings. We also characterise the rings of this class which are Gorenstein, thereby answering a question posed by Li in the toric setting.

#### 8. Joint reduction and adjoint of ideals

Clare D'Cruz (Chennai Mathematical Institute, clare4004@gmail.com)

Using joint reductions of complete ideals, we find formulas for the core and adjoints of the product of complete ideals and formulas for their colengths in a two-dimensional regular local rings. This strengthens a generalization of Briançon–Skoda Theorem due to D. Rees and J. Sally in dimension two. (This is joint work with Saipriya Dubey and J. K. Verma)

#### 9. Observations (and a few results) on infinite free resolutions

David Eisenbud (Mathematical Sciences Research Institute, Berkeley, de@berkeley.edu)

For many years the Kaplansky–Serre problem—"Is the Poincaré series of every local ring rational?"—dominated the subject of infinite free resolutions. It motivated a great deal of work and many positive partial results were obtained, but eventually David Anick showed that the answer in general is "no", leaving the field without a central problem.

Meantime, however, improvements in computation make it possible to look "into" free resolutions in new ways. In work with Cuong, Dao, Kobayashi, Polini and Ulrich (in various combinations) we have noticed some surprising (apparent) regularities that have little to do with Betti numbers. I'll describe the current state of our observations, conjectures, and results.

## 10. Complexity and curvature of (pairs of) Cohen–Macaulay modules, and their applications

Dipankar Ghosh (Indian Institute of Technology Kharagpur, dipankar@maths.iitkgp.ac.in)

The complexity and curvature of a module, introduced by Avramov, measure the growth of Betti and Bass numbers of a module, and distinguish the modules of infinite homological dimension. The notion of complexity was extended by Avramov–Buchweitz to pairs of modules that measure the growth of Ext modules. The related notion of Tor complexity was first studied by Dao. Inspired by these notions, we define Ext and Tor curvature of pairs of modules. The aim of this talk is to study (Ext and Tor) complexity and curvature of pairs of certain CM (Cohen–Macaulay) modules and establish lower bounds of complexity and curvature of pairs of modules in terms of that of a single module. It is well-known that the complexity and curvature of the residue field characterize complete intersection local rings. As applications of our results, we provide some upper bounds of the curvature of the residue field in terms of curvature and multiplicity of any nonzero CM module. As a final upshot, these allow us to characterize

complete intersection local rings (including hypersurfaces and regular rings) in terms of complexity and curvature of pairs of certain CM modules. In particular, under some additional hypotheses, we characterize complete intersection local rings via injective curvature of the ring or that of the module of Kähler differentials. This is joint work with Souvik Dey and Aniruddha Saha.

### 11. Asymptotic regularity of graded families of homogeneous ideals

Tài Huy Hà (*Tulane University*, tha@tulane.edu)

We discuss when the asymptotic regularity of a graded family  $(I_n)_{n\geq 0}$  of homogeneous ideals, i.e., the limit  $\lim_{n\to\infty} \operatorname{reg} I_n/n$ , exists. We consider several cases when this question has an affirmative answer; for example, when the family  $(I_n)_{n\geq 0}$ consists of Artinian ideals, or Cohen–Macaulay ideals of the same codimension over an uncountable base field of characteristic 0, or when its Rees algebra is Noetherian. We provide a combinatorial interpretation of the limit  $\lim_{n\to\infty} \operatorname{reg} I_n/n$  in terms of the associated Newton–Okounkov region in various situations. We give a negative answer to the question of whether the limits  $\lim_{n\to\infty} \operatorname{reg} (I_1^n + \cdots + I_p^n)/n$  and  $\lim_{n\to\infty} \operatorname{reg} (I_1^n \cap \cdots \cap I_p^n)/n$  exist, for  $p \geq 2$  and homogeneous ideals  $I_1, \ldots, I_p$ . We also examine ample evidence supporting a negative answer to the question of whether the asymptotic regularity of the family of symbolic powers of a homogeneous ideal always exists.

### 12. New bounds on Castelnuovo–Mumford regularity of monomial curves and application to sumsets

Le Tuan Hoa (Institute of Mathematics, Vietnam Academy of Science and Technology (VAST), lthoa@math.ac.vn)

A monomial curve C is defined by a sequence of coprime integers  $0 = a_0 < a_1 < \cdots < a_r = d$ . A gap of this sequence is  $a_{i+1} - a_i - 1$ . Gruson–Lazarsfeld–Peskine bound says that  $\operatorname{reg}(C) \leq d - r + 2$ , which is equal to the sum of all gaps plus 2. L'vovsky (1996) showed that it is enough to take the sum of two largest gaps. Under some specific conditions there are several bounds that are better than L'vovsky's bound. In ongoing joint work with D. Q. Tien, using bounds on Frobenius number, we give some new bounds and apply them to studying the structure of sumsets.

### 13. On Herzog's conjecture on cotangent homology of commutative algebras

Srikanth Iyengar (University of Utah, iyengar@math.utah.edu)

This talk is going to be about the cotangent complex of a map of commutative algebras. The focus is on algebras that are essentially of finite type over a field of characteristic zero, and a conjecture of Herzog about the homology of the cotangent complex, that appears in his (soon to be published) manuscript titled "The homological properties of the module of differentials", based on his lectures in Recife, Brazil, in 1980. My goal is to explain where this conjecture is coming from, how it is connected with some other long-standing questions in commutative algebra, and what we know about it at the moment. My talk is based on ongoing joint work with Greg Stevenson and Benjamin Briggs.

#### 14. Upper bounds for second Hilbert coefficients

Mousumi Mandal (Indian Institute of Technology Kharagpur, mousumi@maths.iitkgp.ac.in)

Some upper bounds are given for the second Hilbert coefficient of an  $\mathfrak{m}$ -primary ideal in a Noetherian local ring of dimension at least two, involving sectional genera of the ideal. We characterize the situation when some upper bounds are attained in terms of the depth of the associated graded ring.

## 15. Artinian Gorenstein algebras with a binomial Macaulay dual generator

Shreedevi K. Masuti (Indian Institute of Technology, Dharwad, shreedevi@iitdh.ac.in)

In this talk we will discuss key properties of Artinian Gorenstein K-algebras having binomial Macaulay dual generators. In codimension 3, we show that all such algebras satisfy the strong Lefschetz property, and provide an explicit characterization of when they form a complete intersection.

This is joint work with N. Altafi, R. Dinu, S. Faridi, R. M. Miró-Roig, A. Seceleanu, and N. Villamizar.

#### 16. Symbolic powers of polymatroidal ideals

Somayeh Moradi (Ilam University, so.moradi@ilam.ac.ir)

In this talk, I will explore two conjectures concerning the symbolic powers of polymatroidal ideals. Specifically, for a polymatroidal ideal I, we conjecture that every symbolic power  $I^{(k)}$  is componentwise linear, and that the Castelnuovo–Mumford regularity satisfies reg  $I^{(k)} = \text{reg } I^k$  for all  $k \ge 1$ . I will present several cases in which these conjectures are confirmed. Additionally, I will discuss conditions under which the symbolic and ordinary powers of polymatroidal ideals coincide, thereby providing a proof of the Conforti–Cornuéjols conjecture in the case of matroidal ideals.

#### 17. On toric ideals of weighted oriented graphs

Ramakrishna Nanduri (Indian Institute of Technology Kharagpur, nanduri@maths.iitkgp.ac.in)

Let D = (V(D), E(D), w) be a (vertex) weighted oriented graph and I(D) its edge ideal. In this talk, we present various properties of primitive binomial generators of the toric ideal  $I_D$  of I(D). We classify the circuit binomials of D and their explicit formulas in terms of the minors of the incident matrix of D. We also discuss the robust and strongly robust properties of  $I_D$ . This is joint work with Tapas Kumar Roy.

#### 18. Unmixed torsions and sequentially Cohen–Macaulay modules

Pham Hong Nam (*Thai Nguyen University of Sciences, namph@tnus.edu.vn*)

We characterize the sequential Cohen–Macaulayness of a module M in two ways: (1) in terms of the relationship between numerical invariants of certain module associated with some non-zero Hilbert coefficients and the components in the dimension filtration of M, and (2) in terms of relationship between the unmixed torsion associated with a certain cohomological degree and the arithmetic degree.

# 19. Koszul property and finite linearity defect over g-stretched local rings

Hop D. Nguyen (Institute of Mathematics, Vietnam Academy of Science and Technology (VAST), ndhop@math.ac.vn)

The linearity defect is a measure for the non-linearity of minimal free resolutions of modules over Noetherian local rings. A tantalizing open question due to Herzog and Iyengar asks whether a Noetherian local ring  $(R, \mathfrak{m})$  is Koszul if its residue field  $R/\mathfrak{m}$ has a finite linearity defect. We provide a positive answer to this question when R is a Cohen–Macaulay local ring of almost minimal multiplicity with the residue field of characteristic zero. The proof depends on the study of Noetherian local rings  $(R, \mathfrak{m})$ such that  $\mathfrak{m}^2$  is a principal ideal, which we call *g-stretched* local rings. The class of *g*-stretched local rings subsumes stretched Artinian local rings studied by Sally, and generic Artinian reductions of Cohen–Macaulay local rings of almost minimal multiplicity. An essential part in the proof of our main result is a complete characterization of one-dimensional complete g-stretched local rings. Beside partial progress on Herzog– Iyengar's question, another consequence of our study is a numerical characterization of all g-stretched Koszul rings, strengthening previous work of Avramov, Iyengar, and Şega. This is from joint work with Do Van Kien.

#### 20. Commutative algebra and graph coloring theory

Hidefumi Ohsugi (Kwansei Gakuin University, ohsugi@kwansei.ac.jp)

In this talk, based on the joint works with Mori, Motomura, Shibata, and Tsuchiya, we discuss connections between commutative algebra and graph coloring theory. Let G be a graph on the vertex set  $[n] = \{1, 2, ..., n\}$  with the edge set E(G). A subset  $S \subset [n]$  is called a stable set (or an independent set) of G if  $\{i, j\} \notin E(G)$  for all  $i, j \in S$ with  $i \neq j$ . In particular, the empty set  $\emptyset$  and any singleton  $\{i\}$  with  $i \in [n]$  are stable. Denote  $S(G) = \{S_1, \ldots, S_m\}$  the set of all stable sets of G. Given a subset  $S \subset [n]$ , we associate the (0,1)-vector  $\rho(S) = \sum_{j \in S} \mathbf{e}_j$ . Here  $\mathbf{e}_j$  is the *j*th unit coordinate vector in  $\mathbb{R}^n$ . For example,  $\rho(\emptyset) = (0, \ldots, 0) \in \mathbb{R}^n$ . Let  $K[\mathbf{t}, s] := K[t_1, \ldots, t_n, s]$ be the polynomial ring in n + 1 variables over a field K. Given a nonnegative integer vector  $\mathbf{a} = (a_1, \ldots, a_n) \in \mathbb{Z}_{\geq 0}^n$ , we write  $\mathbf{t}^{\mathbf{a}} := t_1^{a_1} t_2^{a_2} \cdots t_n^{a_n} \in K[\mathbf{t}, s]$ . The stable set ring of G is  $K[G] := \overline{K[\mathbf{t}^{\rho(S_1)}s, \ldots, \mathbf{t}^{\rho(S_m)}s]} \subset K[\mathbf{t}, s]$ . We regard K[G] as a homogeneous algebra by setting each  $\deg(\mathbf{t}^{\rho(S_i)}s) = 1$ . Note that K[G] is a toric ring. Let  $K[\mathbf{x}] := K[x_1, \ldots, x_m]$  denote the polynomial ring in m variables over K with each  $deg(x_i) = 1$ . The stable set ideal  $I_G$  of G is the kernel of the surjective homomorphism  $\pi: K[\mathbf{x}] \to K[G]$  defined by  $\pi(x_i) = \mathbf{t}^{\rho(S_i)} s$  for  $1 \leq i \leq m$ . Then  $I_G$  is a toric ideal, and generated by homogeneous binomials. It turns out that the question of when the stable set ideal is generated by quadratic binomials is linked to the classical graphtheoretical concept of Kempe equivalence. By using this result, we give an algebraic method to examine Kempe equivalence.

#### 21. The first Hilbert coefficient of stretched ideals

Kazuho Ozeki (Nihon University, ozeki.kazuho@nihon-u.ac.jp)

In this talk we explore the almost Cohen–Macaulayness of the associated graded ring of stretched **m**-primary ideals with small first Hilbert coefficient in a Cohen–Macaulay local ring  $(A, \mathbf{m})$ . In particular, we explore the structure of stretched **m**-primary ideals satisfying the inequality  $e_1(I) \leq e_0(I) - \ell_A(A/I) + 4$  where  $e_0(I)$  and  $e_1(I)$  denote the multiplicity and the first Hilbert coefficient respectively.

#### 22. Behrend function and blowup algebras

Claudia Polini (University of Notre Dame, cpolini@nd.edu)

I will survey preliminary results of joint work with Alessio Sammartano and Bernd Ulrich. Given a scheme X of finite type over the complex numbers, the Behrend function, introduced in [1], is a constructible function  $v_X : X(\mathbb{C}) \to \mathbb{Z}$  that allows to compute the degree of the virtual fundamental class of X under suitable assumptions, leading to the solution of numerous problems in enumerative geometry (see e.g. [2, 3]). Even in simple cases though, the Behrend function is very difficult to compute. In this talk I will explain how we compute the Behrend function of arbitrary zero-dimensional monomial ideals in any number of variables and its connections to Rees rings and Rees valuations.

#### REFERENCES

- K. Behrend, Donaldson-Thomas type invariants via microlocal geometry, Ann. of Math. (2) 170 (2009), 1307–1338.
- [2] R. Pandharipande and R. P. Thomas, 13/2 ways of counting curves, Moduli spaces, London Math. Soc. Lecture Note Ser., vol. 411, Cambridge Univ. Press, Cambridge, 2014, pp. 282–333.
- [3] B. Szendroi, Cohomological Donaldson-Thomas theory, String-Math 2014, Proc. Sympos. Pure Math., vol. 93, Amer. Math. Soc., Providence, RI, 2016, pp. 363–396.

### 23. On lengths of modules over certain Artinian complete intersections

Tony J. Puthenpurakal (Indian Institute of Technology Bombay, tputhen@qmail.com)

Let  $(Q, \mathbf{n})$  be a regular local ring of dimension  $c \geq 2$  with algebraically closed residue field  $k = Q/\mathbf{n}$ . Let  $f_1, f_2, \ldots, f_{c-1}, g$  be a regular sequence in Q such that  $f_i \in \mathbf{n}^2$  for all i and  $g \in \mathbf{n}$ . Set  $A = Q/(f_1, \ldots, f_{c-1}, g^r)$  with  $r \geq 2$ . Notice A is an Artinian complete intersection of codimension c. We show that there exists  $\alpha_A \in \mathbb{P}^{c-1}(k)$  such that there exists integer  $m_A \geq 2$  (depending only on A) that divides  $\ell(M)$  for every finitely generated A-module M with  $\alpha_A \notin \mathcal{V}(M)$  (here  $\ell(M)$  denotes the length of Mand  $\mathcal{V}(M)$  denotes the support variety of M). As an application we prove that if kis a field,  $A = k[X_1, \ldots, X_c]/(X_1^{a_1}, \ldots, X_c^{a_c})$  with  $a_i \geq 2$ , and p is a prime number dividing two of the  $a_i$ 's, then p divides  $\ell(E)$  for any A-module E with bounded Betti numbers.

#### 24. On squarefree powers of simplicial trees

Ayesha Asloob Qureshi (Sabanci Univ., ayesha.asloob@sabanciuniv.edu)

Let I be a squarefree monomial ideal. The k-th squarefree power  $I^{[k]}$  of I is the ideal generated by the squarefree monomials among the generators of  $I^k$ . The study of squarefree powers of squarefree monomial ideals is closely connected with the classical theory of matchings in hypergraphs. Moreover, squarefree powers of I provide important information about the ordinary powers of I, since the multigraded minimal free resolution of  $I^{[k]}$  appears as a subcomplex of the multigraded minimal free resolution of  $I^{[k]}$ .

The investigation of squarefree powers of edge ideals of graphs was initiated in two joint works of Herzog, first with Bigdeli and Zaare-Nahandi, and later with Erey, Hibi, and Madani, which establish fundamental results on the connection between matchings of graphs and their edge ideals.

This talk is based on joint work with Elshani Kamberi, Francesco Navarra, and Dharm Veer. We will discuss squarefree powers of facet ideals associated with simplicial trees (equivalently, totally balanced hypergraphs), focusing on the linearity of the minimal free resolutions and the Castelnuovo–Mumford regularity of such ideals.

### 25. Binomial edge ideals and some other related ideals

Sara Saeedi Madani (Amirkabir University of Technology, sarasaeedi@aut.ac.ir)

This is an overview talk mainly on binomial edge ideals of graphs which were introduced by Herzog, Hibi, Hreinsdóttir, Kahle and Rauh in 2010 and at about the same time by Ohtani.

Our special attention is on the results related to the minimal graded free resolution of such ideals. In particular, we discuss the linear strand as well as the Castelnuovo– Mumford regularity of binomial edge ideals.

Furthermore, we briefly talk about some other (related) types of ideals, such as determinantal facet ideals, generalized binomial edge ideals, and Hankel edge ideals of graphs. We also mention some problems that are still open and interesting in the literature.

This talk is mostly based on some joint works with J. Herzog and D. Kiani.

#### 26. Numerical semigroups inspired by Judy Sally

Hema Srinivasan (University of Missouri, srinivasanh@missouri.edu)

We will report on some recent works on a class of numerical semigroups which are called Sally semigroups and some modifications of those. We will describe their minimal generators and construct minimal resolutions for some of them. Classes of numerical semigroups have been studied fixing some of their natural invariants such as the multiplicity, e, embedding dimension, n, or width, w. We study the numerical semigroups of Sally-type with e - w = 1.

## 27. Asymptotic properties for shifted families of numerical semigroups

Dumitru Stamate (Univ. of Bucharest, dumitru.stamate@fmi.unibuc.ro)

Given positive integers  $r_1 < r_2 < \cdots < r_k$ , their associated shifted family consists of the affine semigroups  $M_n = \langle n, n + r_1, \dots, n + r_k \rangle$  for all n > 0. Algebraic properties of the semigroup rings  $K[M_n]$  are commonly explored in terms of the semigroups  $M_n$ .

Answering a conjecture of Herzog and Srinivasan, T. Vu proved that the Betti numbers of  $K[M_n]$  become eventually periodic in n, for n large enough. This opened a path of finding periodic properties for large n. In this talk I will report on joint work with F. Strazzanti which is concerned with the behaviour of a more subtle invariant, the residue, and the nearly Gorenstein property in the shifted family of a numerical semigroup.

#### 28. Symmetric algebras and s-sequences

Zhongming Tang (Suzhou University, zmtang@suda.edu.cn)

Symmetric algebra is an important topic in commutative algebra and algebraic geometry. In this talk, we will introduce the concept of s-sequences, which uses the idea of Gröbner basis, to study some invariants of symmetric algebras. We will mainly discuss the symmetric algebra of the first syzygy module of the maximal ideal of a polynomial ring and certain determinantal ideals.

#### 29. The v-numbers of squarefree monomial ideals

Naoki Terai (Okayama University, terai@okayama-u.ac.jp)

This talk is based on joint work with T. Kataoka and Y. Muta. We express the v-number of a Stanley–Reisner ideal in terms of its Alexander dual complex and prove that the v-number of a cover ideal is just two less than the initial degree of the its syzygy module. We give some relation between the v-number of a Stanley–Reisner ideal and the Serre-depth of the quotient ring of the second symbolic power of the Stanley–Reisner ideal of its Alexander dual. We also show that the v-number of the Stanley–Reisner ideal of a 2-pure simplicial complex is equal to the dimension of its

Stanley–Reisner ring.

### 30. The syzygies of the residue field of Golod rings

Bernd Ulrich (Purdue University, bulrich@purdue.edu)

We provide a recurrence relation for high syzygies of the residue field of Golod rings. For Noetherian local rings of embedding dimension two that are not Artinian complete intersections, we describe all syzygy modules explicitly in terms of their direct sum decomposition into indecomposables. This is joint work with D.T. Cuong, H. Dao, D. Eisenbud, T. Kobayashi, and C. Polini.

#### **31.** Singularities of Herzog varieties

Matteo Varbaro (Università di Genova, varbaro@dima.unige.it)

Let S be a polynomial ring over a field and I a homogeneous ideal. We say that I is a *Herzog ideal* if there exists a monomial order < on S such that  $in_{<}(I)$  is squarefree. In this talk we will discuss Herzog ideals and the projective varieties they define, and will present a recent result with Amy Huang, Jonah Tarasova and Emily Witt affirming that the Herzog smooth projective curves have genus 0. This answers in the case of curves a question raised with Conca some years ago, which later was formulated as conjecture together with Constantinescu and De Negri.

### 32. Joint reductions of complete modules and their mixed Buchsbaum–Rim polynomials

Jugal Verma (Indian Institute of Technology Gandhinagar, jugal.verma@iitgn.ac.in)

We offer new definitions of joint reductions and mixed Buchsbaum–Rim multiplicity for certain collections of modules over a Noetherian local ring and illustrate their application to give two different proofs of a joint-reduction- number-zero theorem for integrally closed modules over two-dimensional regular local rings.

We also relate the mixed Buchsbaum–Rim multiplicity of modules to the Euler– Poincaré characteristic of a natural Koszul complex and relate it to the mixed Buchsbaum– Rim multiplicity of ideals by generalising a lemma from intersection theory.

## 33. Almost Gorenstein and nearly Gorenstein properties of 2dimensional normal rings; using resolution of singularities

Kei-ichi Watanabe (Nihon University, watnbkei@gmail.com)

"Which Cohen-Macaulay rings are near to Gorenstein rings?" is the question that has challenged many commutative algebraists. Goto-Takahashi-Taniguchi (= Endo) proposed "almost Gorenstein rings" and Herzog-Hibi-Stamate proposed "nearly Gorenstein" rings. In this talk, I will explain a method to compute canonical trace ideal of a 2-dimensional rational singularities, using resolution of singularities. It turns out that non-Gorenstein nearly Gorenstein rings are rather few among 2-dimensional normal local rings, while all 2-dimensional rational and elliptic singularities are almost Gorenstein.

This talk is based on joint work in progress with Tomohiro Okuma and Ken-ichi Yoshida.

## 34. Generalized binomial edge ideals and the Cartwright–Sturmfels property

Volkmar Welker (Philipps-Universität Marburg, welker@mathematik.uni-marburg.de)

This is joint work with Aldo Conca and Emanuela De Negri.

A  $\mathbb{Z}^n$ -graded ideal I in a polynomial ring S over a field is Cartwright–Sturmfels if its multigraded generic initial ideal is radical.

In this talk we associate to a simple undirected graph G = ([n], E) and a number m an ideal  $I_G(m)$ . The ideal is generated by those 2-minors of an  $m \times n$ -matrix  $(x_{ij})_{i \in [m], j \in [n]}$  of indeterminates which arise by selecting two arbitrary rows and two columns j, k for which  $\{j, k\}$  is an edge in E.

We show that  $I_G(m)$  is Cartwright–Sturmfels for all m and G. For the complete graph  $G = K_n$  or for m = 2 this is a known result from work by Conca, Gorla and De Negri.

We also provide examples which show that in general an analogous result for 3minors associated to a 3-uniform hypergraph is false. We give necessary conditions on the hypergraph which imply the Cartwright–Sturmfels property.

#### 35. Generalized binomial edge ideals of bipartite graphs

Guangjun Zhu (Soochow University, zhuguangjun@suda.edu.cn)

Connected bipartite graphs whose binomial edge ideals are Cohen–Macaulay have been classified by Bolognini et al. In this talk, we compute the depth, Castelnuovo– Mumford regularity, and dimension of the generalized binomial edge ideals of these graphs.

# **III. ABSTRACTS OF POSTERS**

1. Canonical module and test module of (extended) Rees algebra Rahul Ajit (University of Utah, rahulajit@math.utah.edu)

Various properties (Cohen–Macaulay, Gorenstein, rationality...) of associated graded ring, Rees and extended Rees algebra have been studied extensively by many mathematicians. Here, we give a decomposition of test module of (extended) Rees algebra and answer a conjecture of Hara–Watanabe–Yoshida on F-rationality in full generality. Using different techniques, very recently Koley–Kummini and Kotal–Kummini obtained similar results in a restricted setup.

#### 2. The v-number of binomial edge ideals

Siddhi Balu Ambhore (Indian Institute of Technology Gandhinagar, siddhi.ambhore@iitgn.ac.in)

The invariant v-number was introduced very recently in the study of Reed–Mullertype codes. Jaramillo and Villarreal started the study of the v-number of edge ideals. Inspired by their work<sup>1</sup>, we have initiated the study of the v-number of binomial edge ideals. We have studied some properties and the bounds of the v-number of binomial edge ideals. We have explicitly found the v-number of binomial edge ideals locally at the associated prime ideal corresponding to the cut set  $\emptyset$ . We have shown that the v-number of Knutson binomial edge ideals is less than or equal to the v-number of their initial ideals. We have classified all binomial edge ideals whose v-number is 1. We have further tried to relate the v-number with the Castelnuovo–Mumford regularity of binomial edge ideals and have given a conjecture in this direction.

This is joint work with Kamalesh Saha and Indranath Sengupta.

## 3. Symmetric decomposition of the Hilbert function of an ideal

Meghana Bhat (Indian Institute of Technology Dharwad, 212071001@iitdh.ac.in)

Let  $(R, \mathfrak{m})$  be a local ring over a field k and J an ideal in R such that A = R/J is an Artinian Gorenstein (AG) k-algebra. In 1989, A. Iarrobino introduced the symmetric decomposition of the Hilbert function of A. This became a very powerful tool for classifying the Hilbert functions of AG k-algebras. This poster introduces the symmetric decomposition of the Hilbert function of any ideal I in A. Our hope is that this result will be useful in classifying the possible Hilbert function of an ideal in an AG k-algebra. We illustrate this by giving a complete list of 2-admissible sequences of length at most

<sup>&</sup>lt;sup>1</sup>Jaramillo, D.; Villarreal, R. H., *The* v-*number of edge ideals*, J. Combin. Theory Ser. A **177** (2021), 105310, 35 pp.

3 starting with  $h_0 = 2$  that are realizable by an ideal in an AG k-algebra.

## 4. Complexity, curvature and homological dimension of modules under linkage

Subhadip Bhowmick (Indian Institute of Technology Kharagpur, sbhowmick712@kgpian.iitkgp.ac.in)

We analyze how (projective and injective) complexity, curvature, and complete intersection dimension behave under linkage of modules and ideals. Let R be a Gorenstein local ring. Consider a Gorenstein perfect ideal  $\mathfrak{a}$  (e.g.,  $\mathfrak{a}$  is generated by an R-regular sequence). Let M and N be two Cohen-Macaulay R-modules linked by  $\mathfrak{a}$ . We prove that  $\operatorname{cx}_R(M) = \operatorname{inj} \operatorname{cx}_R(N)$  and  $\operatorname{curv}_R(M) = \operatorname{inj} \operatorname{curv}_R(N)$ . In particular, when R is complete intersection,  $\operatorname{cx}_R(M) = \operatorname{cx}_R(N)$  and  $\operatorname{curv}_R(M) = \operatorname{curv}_R(N)$ . Furthermore, we show that  $\operatorname{pd}_R(M) = \operatorname{pd}_R(N)$  and  $\operatorname{CI-dim}_R(M) = \operatorname{CI-dim}_R(N)$ . If any of these dimensions is finite, it is equal to  $\operatorname{ht}(\mathfrak{a})$ . Similar results are obtained for linkage of ideals. All these results highly extend a classical result of Peskine and Szpiro in many directions. We construct several examples that complement our results. These also show how properties like 'integrally closed', ' $\mathfrak{m}$ -full' and 'Burch' behave under linkage of ideals. This is joint work with Dipankar Ghosh.

## 5. On the strong persistence property of path and cover ideals of some graphs

Hafsa Bibi (Institute of Technology Bandung, hafsaliaqat600@gmail.com)

Monomial ideals play an important role in combinatorial commutative algebra. In a commutative Noetherian ring R, the associated primes are connected to the primary decomposition of ideals. There are some known classes of monomial ideals that satisfy the strong persistence property. These classes include: edge ideals of simple graphs, graphs with loops; vertex cover ideals of perfect graphs, cycle graphs of odd orders, and wheel graphs of even orders; and irreducible ideals. In a polynomial ring R, I explore new classes of monomial ideals to understand the strong persistence property. Specifically, examine the path ideal of length 2 for various centipede-related graphs, the path ideal of line graphs, and the cover ideal of the sunlet graph (an imperfect graph). The results show that these ideals satisfy the strong persistence property. Additionally, compute the index of stability and describe the stable set of associated primes of the cover ideal of different graphs.

### 6. Asymptotic behaviour and stability index of v-numbers of

#### graded ideals

Prativa Biswas (Indian Institute of Technology Kharagpur, prativabiswassnts@gmail.com)

Let I be a graded ideal in  $S = K[x_1, \ldots, x_n]$ . By v(I), we denote the v-number of I. Recently, Ficarra and Sgroi initiated the study of v-numbers of powers of graded ideals. They proved that for a graded ideal I,  $v(I^k)$  is a linear function in k for  $k \gg 0$ . Later, Ficarra conjectured that if I is a monomial ideal with linear powers, then  $v(I^k) = \alpha(I)k - 1$  for all  $k \ge 1$ , where  $\alpha(I)$  denotes the initial degree of I. We generalize this conjecture for graded ideals and prove it for several classes of graded ideals: principal ideals, ideals I with depth(S/I) = 0, cover ideals of graphs, t-path ideals, monomial ideals generated in degree 2, edge ideals of weighted oriented graphs. We simplify the conjecture for several classes of graded ideals (including square-free monomial ideals) by showing it is enough to prove the conjecture for k = 1 only. Additionally, we define the stability index of the v-number for graded ideals and investigate this index for edge ideals of graphs.

#### 7. Support-2 monomial ideals that are Simis

Paromita Bordoloi (Indian Institute of Technology, Jammu, 2022rma0026@iitjammu.ac.in)

A monomial ideal  $I \subseteq \mathbb{K}[x_1, \ldots, x_n]$  is called a Simis ideal if  $I^{(s)} = I^s$  for all  $s \geq 1$ , where  $I^{(s)}$  denotes the s-th symbolic power of I. Let I be a support-2 monomial ideal such that its irreducible primary decomposition is minimal. We prove that I is a Simis ideal if and only if  $\sqrt{I}$  is Simis and I has standard linear weights. This result thereby proves a recent conjecture for the class of support-2 monomial ideals proposed by Mendez, Pinto, and Villarreal. Furthermore, we give a complete characterization of the Cohen–Macaulay property for support-2 monomial ideals whose radical is the edge ideal of a whiskered graph. Finally, we classify when these ideals are Simis in degree 2.

## 8. Regularity of integral closures of equigenerated monomial ideals in three variables

Yijun Cui (Soochow (Suzhou) University, 237546805@qq.com)

Let  $I \subset S = K[x_1, x_2, x_3]$  be a monomial ideal generated in degree d. In our research, we proved that  $\operatorname{reg}(\overline{I}) \leq \operatorname{reg}(I)$ . In particular, we proved  $\overline{I} = I$  when I =

 $I_1 + I_2$ , where  $I_i = x_1^{a_i}(x_2^{b_{i,1}}x_3^{c_{i,1}}, \dots, x_2^{b_{i,\ell_i}}x_3^{c_{i,\ell_i}})$  for  $i \in [2]$  and  $a_1 < a_2$ .

### 9. Admissible matchings and the Castelnuovo–Mumford regularity of square-free powers

Kanoy Kumar Das (Chennai Mathematical Institute, India, kanoydas0296@gmail.com)

Let I be any square-free monomial ideal, and  $\mathcal{H}_I$  denote the hypergraph associated with I. Refining the concept of k-admissible matching of a graph defined by Erev and Hibi, we introduce the notion of *generalized k-admissible matching* for any hypergraph. Using this, we provide a sharp lower bound on the (Castelnuovo–Mumford) regularity of  $I^{[k]}$ , where  $I^{[k]}$  denotes the  $k^{\text{th}}$  square-free power of I. In the special case when I is equigenerated in degree d, this lower bound can be described using a combinatorial invariant  $\operatorname{aim}(\mathcal{H}_I, k)$ , called the *k*-admissible matching number of  $\mathcal{H}_I$ . Specifically, we prove that  $\operatorname{reg}(I^{[k]}) \geq (d-1)\operatorname{aim}(\mathcal{H}_I, k) + k$ , whenever  $I^{[k]}$  is non-zero. Even for the edge ideal I(G) of a graph G, it turns out that aim(G, k) + k is the first general lower bound for the regularity of  $I(G)^{[k]}$ . In fact, when G is a forest, aim(G,k) coincides with the k-admissible matching number introduced by Erev and Hibi. We discuss two instances where this general lower bound is attained. We show that if G is a block graph, then  $\operatorname{reg}(I(G)^{[k]}) = \operatorname{aim}(G, k) + k$ , and this result can be seen as a generalization of the corresponding regularity formula for forests. Additionally, for a Cohen-Macaulay chordal graph G, we prove that  $\operatorname{reg}(I(G)^{[2]}) = \operatorname{aim}(G,2) + 2$ . Finally, we propose a conjecture on the regularity of square-free powers of edge ideals of chordal graphs.

#### 10. Density functions of filtrations of homogeneous ideals

Suprajo Das (IIT Madras, dassuprajo@gmail.com)

Let R be a standard graded Noetherian domain over a field. Let  $\{I_n\}$  be a (not necessarily Noetherian) filtration of homogeneous ideals in R. We shall associate a continuous function, called the *density function*, which keeps track of the growth of the graded pieces of  $I_n$  systematically. This is motivated by an earlier construction of the Hilbert–Kunz density function due to Trivedi. We further show that this density function is continuously differentiable when we deal with saturated filtrations. The equality of density functions will also be discussed. This poster will be based on two ongoing joint projects: (1) with S. Roy and V. Trivedi, and (2) with S. Roy and H. L. Truong.

### 11. On the unmixed and Cohen–Macaulay parity binomial edge

#### ideals of chordal graphs

Deblina Dey (IIT Madras, deblina.math@gmail.com)

In this poster, we characterize all unmixed and Cohen–Macaulay parity binomial edge ideals of chordal graphs in terms of the graph structure. We introduce an algorithm for chordal graphs to construct maximal trees along with certain disconnector sets, and show that these structures interact well with the unmixedness property. This is a joint work with A. V. Jayanthan and Sarang Sane.

## 12. Quasilifting of hulls and depth of tensor product of modules

Sutapa Dey (Indian Institute of Technology Hyderabad, ma20resch11002@iith.ac.in)

A quasilift type construction is used to obtain some bounds on the depth of the tensor product of certain modules over a local  $\mathcal{TE}$  ring. In the process, we recover a result of Celikbas, Sadeghi and Takahashi for local complete intersection rings.

## 13. Cochordal zero divisor graphs and Betti numbers of their edge ideals

Le Xuan Dung (Hong Duc University, Thanh Hoa, Vietnam, lexuandung@hdu.edu.vn)

This is joint work with Vu Q. Thanh. We associate a sequence of positive integers, termed the *type sequence*, with a cochordal graph. Using this type sequence, we compute all graded Betti numbers of its edge ideal. We then classify all positive integer n such that the zero divisor graph of  $\mathbb{Z}/n\mathbb{Z}$  is cochordal and determine all the graded Betti numbers of its edge ideal.

## 14. The canonical trace of Cohen–Macaulay algebras of codimension 2

Antonino Ficarra (Basque Center for Applied Mathematics (BCAM), aficarra@bcamath.org)

In this poster, we investigate a conjecture of Jürgen Herzog. Let S be either a regular local ring with residue field K or a positively graded K-algebra,  $I \subset S$  be a perfect ideal of grade 2, and let R = S/I with canonical module  $\omega_R$ . Jürgen Herzog conjectured that the canonical trace  $\operatorname{tr}(\omega_R)$  is obtained by specialization from the generic case of maximal minors. The conjecture is established in several cases; for instance, when R is generically Gorenstein. This latter result is applied to numerical semigroup rings generated by three elements, and to perfect generic monomial ideals of height 2. The perfect monomial ideals of height 2 are classified in terms of a certain graph and certain integer sequences attached to the ideal. As the final conclusion of all of our results, the nearly Gorenstein monomial ideals of height 2 are classified.

#### 15. Affine modifications in cancellation and embedding problems

Parnashree Ghosh (*Tata Institute for Fundamental Research, ghoshparnashree@gmail.com*)

We will explore two fundamental problems in affine geometry: the "Embedding problem" and the "Cancellation problem". The affine modification technique, introduced by Kaliman and Zaidenberg, has played a crucial role in achieving significant breakthroughs in these areas. In this presentation, we will discuss recent advancements in these problems through the application of this construction. Specifically, we will outline a general framework that produces various families of hyperplanes satisfying the "Abhyankar–Sathaye embedding conjecture" and introduces a new class of counterexamples to the "Zariski cancellation problem" in positive characteristic.

## 16. Zariski's finiteness theorem and properties of some rings of invariants

Buddhadev Hajra (Chennai Mathematical Institute, hajrabuddhadev92@gmail.com)

We have found a short proof, using a new idea, of an important special case of Zariski's result about the finite generation in connection with the famous Hilbert's Fourteenth Problem. This result is useful for invariant subrings of unipotent or connected semisimple groups. Our proof is a combination of commutative algebra, algebraic geometry and algebraic topology. Additionally, we will present a generalization of a well-known result by Andrzej Tyc. Our result proves that the quotient space under a regular  $\mathbf{G}_a$ -action on an affine space over the field of complex numbers has at most rational singularities, under an assumption about the quotient morphism. I will also sketch the main idea of the proof of a result that is analogous to Masayoshi Miyanishi's result for the invariant ring of a  $\mathbf{G}_a$ -action on the polynomial ring R[X, Y, Z], where Ris an affine Dedekind domain. This proof involves some classical topological methods. The content of this poster is based on joint work with R.V. Gurjar and S.R. Gurjar.

### 17. Symbolic powers of path ideals of simple graphs

Ritam Halder (Indian Institute of Technology Kharagpur,

#### ritamhalder10@gmail.com)

This poster contains my recent joint work with Arindam Banerjee. We studied four path ideals for which the symbolic and ordinary powers coincide. We also gave some generalized results for *n*-path ideals. We gave a necessary condition for any *n*-path ideal to have identical ordinary and symbolic powers. Also we introduced the *k*-uniform Erdös-Rényi random hypergraph and showed under some probability regime w.h.p. for *k*-uniform Erdös-Rényi random hypergraphs the ordinary and symbolic powers do not coincide.

#### 18. On some rings of differentiable type

Sayed Sadiqul Islam (IIT Bombay, 22d0786@iitb.ac.in)

Let K be a field of characteristic 0 and  $S = K[x_1, \ldots, x_r]/I$  an affine domain over K. Let  $R = S_P$  where  $P \in \text{Spec}(S)$  such that R is regular. Let F be any quasicoefficient field of R containing K. We show that  $D_F(R)$ , the ring of F-linear differential operators on R is left and right Noetherian and have finite global dimension. We also prove similar result for  $R^h$ , the Henselization of R. As an application we prove that  $D_F(R)/D_F(R)P \cong E(\kappa(P))$ , where  $E(\kappa(P))$  is the injective hull of residue field of R.

#### 19. Linear quotients of connected ideals of graphs

Omkar Deepak Javadekar (IIT Bombay, omkarjavadekar@gmail.com)

We study a higher analogue of the edge ideal, called the *t*-connected ideal  $J_t(G)$ , which is generated by monomials corresponding to connected subsets of size t in a graph G. This generalizes the edge ideal, which arises when t = 2. Our main results establish conditions for  $J_t(G)$  to have a linear resolution. We show that for chordal graphs,  $J_t(G)$  has a linear resolution if and only if the graph is *t*-gap-free, and that this is equivalent to  $J_t(G)$  having linear quotients. We further prove that if G is gap-free and *t*-claw-free, then  $J_t(G)$  has linear quotients and, consequently, a linear resolution. In addition, our results recover some of the known results on the linearity of resolution of t-path ideals of graphs. The poster is based on joint work with H. Ananthnarayan and Aryaman Maithani.

#### REFERENCE

H. Ananthnarayan, Omkar Javadekar, and Aryaman Maithani, *Linear quotients of con*nected ideals of graphs, accepted for publication in the Journal of Algebraic Combinatorics.

#### 20. Closed subsets defined by annihilators of Ext modules

Kaito Kimura (Nagoya University, m21018b@math.nagoya-u.ac.jp)

It is classically known, via Nagata's criterion, that the singular locus and the non-Gorenstein locus of a reasonably nice ring are closed subsets. Iyengar and Takahashi [3] introduced the notion of cohomology annihilators and proved that it is a defining ideal of the singular locus under mild assumptions. The cohomology annihilator of a ring R is the ideal consisting of elements a such that there exists an integer n with  $a \operatorname{Ext}_{R}^{n}(M, N) = 0$  for all finitely generated modules M, N. They also discovered a relationship between the behavior of the cohomology annihilator and the strong generation of the module and derived categories. Their results raise the natural question: Which Ext modules have annihilator ideals being equal to the defining ideal of the singular locus?

By the method of Dey and Takahashi [2], the above question for the singular locus can be reduced to a similar question for the non-Gorenstein locus. When R is a Cohen–Macaulay local ring of dimension d with canonical module, Dao, Kobayashi, and Takahashi [1] show that the ideal consisting of elements a such that  $a \operatorname{Ext}_{R}^{d+1}(M, R) = 0$ for any finitely generated module M coincides with the trace of the canonical module, and hence is a defining ideal of the non-Gorenstein locus. In this poster, we provide a non-Cohen–Macaulay analog of this result, thereby explicitly describing the defining ideals of the singular locus and the non-Gorenstein locus in terms of annihilators of Ext modules. This poster is based on [4].

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## 21. Frobenius Betti numbers for graded Cohen–Macaulay rings of finite type

Nirmal Kotal (Institute of Mathematical Sciences Chennai, nirmalk@imsc.res.in)

Frobenius Betti numbers provide a powerful homological invariant that refines the Hilbert–Kunz multiplicity and offers deep insight into the nature of singularities in rings of positive characteristic. Despite their conceptual importance, explicit computations of these invariants are known in only a handful of cases. In this poster, we focus on the classifications of Cohen–Macaulay graded rings of finite type and provide explicit computations of their Frobenius Betti numbers. Our results highlight a rich interplay between the structure of maximal Cohen–Macaulay modules, the Frobenius endomorphism, and homological dimensions. This study not only broadens the landscape of known examples but also points toward deeper connections between singularity theory and the homological behavior of the Frobenius functor.

#### 22. Koszul complexes and Tate resolutions

Ganapathy Krishnamoorthy (Indian Institute of Technology Madras, ganapathy.math@gmail.com)

Over a commutative Noetherian ring R, the Koszul complex on a sequence of elements  $\boldsymbol{x} = x_1, \ldots, x_n$  encodes several properties of the ideal  $(\boldsymbol{x})$ . The Koszul complex on  $\boldsymbol{x}$  is a free resolution of  $R/(\boldsymbol{x})$  if and only if  $\boldsymbol{x}$  is a regular sequence. When the Koszul complex is not a resolution, one has the Tate resolution, which is obtained by killing the non-zero homology modules of the Koszul complex in an inductive manner.

In joint work with Sarang Sane, we show that there exists a chain complex (DG-algebra) map from the Tate resolution on the powers of  $\boldsymbol{x}$  to the Koszul complex on  $\boldsymbol{x}$ . We also provide some applications of this result.

#### 23. The slope of the v-function and the Waldschmidt constant

Manohar Kumar (Indian Institute of Technology Kharagpur, manhar349@gmail.com)

In this paper, we study the asymptotic behaviour of the v-number of a Noetherian graded filtration  $\mathcal{I} = \{I_{[k]}\}_{k\geq 0}$  of a Noetherian  $\mathbb{N}$ -graded domain R. Recently, it was shown that  $v(I_{[k]})$  is periodically (or quasi-) linear in k for  $k \gg 0$ . We show that all

these linear functions have the same slope, i.e.  $\lim_{k\to\infty} \frac{v(I_{[k]})}{k}$  exists, which is equal to

 $\lim_{k\to\infty} \frac{\alpha(I_{[k]})}{k}$ , where  $\alpha(I)$  denotes the minimum degree of a non-zero element in I. In particular, for any Noetherian symbolic filtration  $\mathcal{I} = \{I^{(k)}\}_{k\geq 0}$  of R, it follows that  $\lim_{k\to\infty} \frac{\alpha(I^{(k)})}{k} = \hat{\alpha}(I)$ , the Waldschmidt constant of I. Next, for a non-equigenerated square-free monomial ideal I, we prove that

$$v(I^{(k)}) \le \operatorname{reg}(R/I^{(k)})$$

for  $k \gg 0$ . Also, for an ideal I having the symbolic strong persistence property, we give a linear upper bound on  $v(I^{(k)})$ . As an application, we derive some criteria for a square-free monomial ideal I to satisfy  $v(I^{(k)}) \leq \operatorname{reg}(R/I^{(k)})$  for all  $k \geq 1$ , and provide several examples in support. In addition, for any simple graph G with the cover ideal J(G), we establish that

$$v(J(G)^{(k)}) \le \operatorname{reg}(R/J(G)^{(k)})$$

for all  $k \ge 1$ , and  $v(J(G)^{(k)}) = \operatorname{reg}(R/J(G)^{(k)}) = \alpha(J(G)^{(k)}) - 1$  for all  $k \ge 1$  if and only if G is a Cohen–Macaulay very-well covered graph.

#### 24. $\phi$ -conducive rings

Rahul Kumar (Birla Institute of Technology and Science Pilani, kumar.rahul@pilani.bits-pilani.ac.in)

Let  $\mathcal{H}$  be the set of all commutative rings with unity whose nilradical is a divided prime ideal. If  $R \in \mathcal{H}$ , then there is a ring homomorphism  $\phi : \operatorname{tq}(R) \to R_{\operatorname{Nil}(R)}$  given by  $\phi(r/s) = r/s$  where  $r \in R$  and  $s \in R \setminus Z(R)$ . A ring  $R \in \mathcal{H}$  is said to be a  $\phi$ -conducive ring if for each overring T of  $\phi(R)$ , other than  $R_{\operatorname{Nil}(R)}$ ,  $\operatorname{Nil}(\phi(R)) \subset (\phi(R) : T)$ . We study various properties of a  $\phi$ -conducive ring.

### **25.** Rees algebras and $G_{d-2}$ condition

Suraj Kumar (Indian Institute of Technology Delhi, Suraj.Kumar@maths.iitd.ac.in)

Let  $R = k[x_1, \ldots, x_d]$  be a polynomial ring over a field k and  $I = (f_1, \ldots, f_{d+1})$  be a height two perfect ideal that is linearly presented. Furthermore, we suppose that the ideal I satisfies  $G_{d-2}$  but satisfies neither  $G_{d-1}$  nor  $G_d$ . The  $G_s$  bounds the minimal number of generators of  $I_p$  by the height of  $\mathfrak{p}$  for all primes  $\mathfrak{p}$  up to height s-1. In this setting, we provide explicit formulas for the defining ideal of the Rees algebra of I, denoted by R(I). We further demonstrate that R(I) is not always a Cohen-Macaulay ring. While the attempts to find the defining ideal of the Rees algebra of ideals satisfying  $G_d$  or  $G_{d-1}$  conditions are well documented, the  $G_{d-2}$  case is still unexplored.

## 26. Application of Betti splittings to the regularity of binomial edge ideals

Paramhans Kushwaha (Indian Institute of Technology Jammu, 2022rma2004@iitjammu.ac.in)

Betti splitting is an important tool to study some of the invariants of an ideal such as projective dimension, Castelnuovo–Mumford regularity etc. In this poster session, we will prove an upper bound for the binomial edge ideals of trees using Betti splitting. Also, we provide a lower bound for the regularity. As a consequence, we give the exact regularity for binomial edge ideals for certain classes of trees.

#### 27. Ideals generated by some minors of a generic $m \times n$ matrix

Vivek Bhabani Lama (Indian Institute of Technology Kharagpur, vivekbhabanilama@kgpian.iitkgp.ac.in)

We characterize the primality of ideals generated by some 2-minors of a  $3 \times n$  matrix in terms of the row and column graphs of these ideals under the condition of graph connectivity. This generalizes a well-known result in the case of binomial edge ideals. This is joint work with Arindam Banerjee and Kanoy Kumar Das.

# 28. Popescu-type approximation of complete local rings and applications

Shiji Lyu (University of Illinois, slyu@uic.edu)

For concreteness, let k be a field of characteristic 0 and R be the power series ring  $k[[T_1, \ldots, T_n]]$ . Finite type schemes over R were used in, for example, Hironaka's resolution of singularities and work of de Fernex-Ein-Mustata on ACC of log canonical thresholds. We discuss a systematic way of approximating finite type schemes over Rusing schemes essentially of finite type over k, preserving various types of singularities and homological properties. This allows us to extend known results and constructions for varieties to finite type schemes over R, including formulas for multiplier ideals, deformation of singularities, and big Cohen-Macaulay algebras. Our construction works in arbitrary characteristic, and has applications to étale cohomology of rigid analytic spaces, and more. This is joint work in preparation with Shizhang Li and Bogdan Zavyalov.

## 29. On a graph with respect to the idempotents of a commutative ring

Praveen Mathil (Birla Institute of Technology and Science Pilani, maithilpraveen@gmail.com)

Let R be a ring with unity. The idempotent graph  $G_{Id}(R)$  of a ring R is an undirected simple graph whose vertices are the set of all the elements of ring R and two vertices x and y are adjacent if and only if x + y is an idempotent element of R. Razaghi et al.<sup>2</sup> studied basic properties of  $G_{Id}(R)$  such as connectedness, diameter and girth. In this poster, first, we correct a result obtained by Razaghi et al. and determine the precise structure of the idempotent graph of a local ring. Further, we obtain a necessary and sufficient condition on the ring R such that  $G_{Id}(R)$  is planar. We prove that  $G_{Id}(R)$  is an outerplanar graph if and only if R is a local ring. Moreover, we classify all the finite commutative rings R such that  $G_{Id}(R)$  is a claw-free, cograph, split graph and threshold graph, respectively. We conclude that for a finite non-local commutative ring, the latter two graph classes of  $G_{Id}(R)$  are equivalent if and only if R is a Boolean ring.

## **30.** Standard multigraded Hibi rings and Cartwright–Sturmfels ideals

Koji Matsushita (Univ. of Tokyo, koji-matsushita@g.ecc.u-tokyo.ac.jp)

Let L be a finite lattice and k be a field. Let  $S_L = k[x_\alpha : | \alpha \in L]$  be a polynomial ring over k and  $I_L$  be an ideal of  $S_L$  generated by

 $\mathcal{F}_L := \{ f_{\alpha,\beta} : \alpha, \beta \in L \text{ are incomparable} \},\$ 

where  $f_{\alpha,\beta} := x_{\alpha}x_{\beta} - x_{\alpha\wedge\beta}x_{\alpha\vee\beta}$  for incomparable elements  $\alpha, \beta \in L$ . A compatible monomial order is a monomial order  $\preceq$  on  $S_L$  such that  $\operatorname{in}_{\preceq}(f_{\alpha,\beta}) = x_{\alpha}x_{\beta}$  for all incomparable elements  $\alpha, \beta \in L$ . It is known that the followings are equivalent ([2], see also [1, Chapter 6]):

(1)  $I_L$  is a prime ideal;

- (2) L is a distributive lattice;
- (3)  $\mathcal{F}_L$  is a Gröbner basis of  $I_L$  with respect to a compatible monomial order.

 $<sup>^{2}</sup>A$  graph with respect to idempotents of a ring. J. Algebra Appl., **20**(6): Paper No. 2150105, 8, 2021.

When the equivalent condition holds, the residue ring  $S_L/I_L$  is called a *Hibi ring*.

Usually, the Hibi ring  $S_L/I_L$  is regarded as a standard Z-graded ring by setting  $\deg(x_{\alpha}) = 1$ . However, in this poster presentation, we consider endowing the Hibi ring with a standard multigraded ring structure. In particular, we address the following problems:

- How should we equip  $S_L$  with a standard multigrading so that  $I_L$  is homogeneous?
- Compute the Hilbert series of  $S_L/I_L$ .
- Under what conditions is  $I_L$  a Cartwright–Sturmfels ideal?

We will give answers to the above questions.

This poster presentation is based on joint work with Koichiro Tani.

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#### 31. Analysis in the calculation of Hilbert–Kunz multiplicity

Cheng Meng (Yau Mathematical Sciences Center, Jingzhai, Tsinghua University, cheng319000@tsinghua.edu.cn)

We present theoretical and computational results concerning the h-function, a numerical invariant for local rings of characteristic p that recovers several important invariants, including the Hilbert–Kunz multiplicity, F-signature, F-threshold, and F-signature of pairs with principal divisors.

The main result in the theoretical part consists of a novel integration formula for the h-function of hypersurfaces defined by polynomials of the form  $\phi(f_1, \ldots, f_s)$ , where  $\phi$  is a polynomial and  $f_i$  are polynomials in independent sets of variables. A key example we analyze is the diagonal hypersurface  $x_1^{d_1} + \cdots + x_s^{d_s}$ , which serves as a fundamental case for our results. In the computational part, we demonstrate applications of this integration formula through three key results: we establish the asymptotic behavior of the Hilbert–Kunz multiplicity for Fermat hypersurfaces of degree 3, extending the degree 2 case previously resolved by Gessel and Monsky. We prove an inequality conjectured by Watanabe and Yoshida for all odd primes, generalizing Trivedi's prior work that

was limited to the case of sufficiently large primes. We further generalize an inequality initially established by Caminata, Shideler, Tucker, and Zerman, extending its validity to almost all primes.

### 32. On upper bounds for the dimension of the singularity category of a commutative ring

Yuki Mifune (Nagoya University, yuki.mifune.c9@math.nagoya-u.ac.jp)

Let R be a commutative Noetherian ring. Denote by mod R the category of finitely generated R-modules, by  $D^b(R)$  the bounded derived category of mod R. The singularity category, introduced by Buchweitz [1], is defined as the Verdier quotient of  $D^b(R)$ by the category of perfect complexes over R; that is,

$$D_{sg}(R) = D^b(R) / \text{thick}R.$$

This category reflects the singularity of R in the sense that  $D_{sg}(R)$  is trivial if and only if R is regular.

For an essentially small triangulated category  $\mathcal{T}$ , we can define the Rouquier dimension of  $\mathcal{T}$ , denote by dim  $\mathcal{T}$ . This invariant measures how many times one needs to take mapping cones starting from a single object to generate the entire category. Concerning upper bounds for the Rouquier dimension of  $D_{sg}(R)$ , Liu [3] obtained the following result.

**Theorem (Liu).** Let  $(R, \mathfrak{m}, k)$  be an equicharacteristic excellent local ring with an isolated singularity, and I an  $\mathfrak{m}$ -primary ideal of R contained in the annihilator of  $D_{sq}(R)$ . Then one has

$$\dim \mathcal{D}_{sg}(R) \le (\mu(I) - \operatorname{depth} R + 1)\ell\ell(R/I) - 1.$$

Here, the annihilator of  $D_{sg}(R)$  is defined as the set of elements  $r \in R$  such that r annihilates the endomorphism rings of all objects in  $D_{sg}(R)$ . We denote by  $\mu(I)$  the minimal number of generators of the ideal I, and by  $\ell\ell(R/I)$  the Loewy length of R/I. In the case where R is Cohen-Macaulay, the theorem was proved by Dao and Takahashi [2]. Our main result generalizes a theorem of Liu to arbitrary commutative Noetherian rings.

This poster is based on the preprint [4].

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#### 33. When do pseudo-Gorenstein become Gorenstein?

Sora Miyashita (University of Osaka, u804642k@ecs.osaka-u.ac.jp)

We discuss the relationship between the trace ideal of the canonical module and pseudo-Gorensteinness. In particular, under certain mild assumptions, we show that every positively graded domain that is both pseudo-Gorenstein and nearly Gorenstein is Gorenstein. As an application, we clarify the relationships among nearly Gorensteinness, almost Gorensteinness, and levelness—notions that generalize Gorensteinness—in the context of standard graded domains. Moreover, we give a method for constructing quasi-Gorenstein rings by taking a Veronese subalgebra of certain Noetherian graded rings.

#### 34. Dominating ideals and closed neighborhood ideals of graphs

Aslı Tuğcuoğlu Musapaşaoğlu (Sabancı University, Turkey, atmusapasaoglu@sabanciuniv.edu)

In comparison to the edge ideals and cover ideals associated with graphs which are well-known and extensively studied, relatively little is known in the case of closed neighborhood ideals and dominating ideals of graphs. In this poster, we present several results concerning the closed neighborhood ideals and the dominating ideals of graphs, specifically focusing on certain classes of trees and cycles, as well as the dominating ideals of path graphs. We investigate whether these ideals are normally torsion-free, and if they are not, we inquire whether they exhibit the persistence property for some classes. Additionally, we demonstrate the componentwise linearity of the dominating ideals of path graphs by describing a linear quotient order for their minimal generating sets, and we provide formulas for their Betti numbers, regularity, and projective dimension.

#### 35. Binomial ideals and rook polynomials of polynomials

Francesco Navarra (Sabanci University, Turkey, francesco.navarra@sabanciuniv.edu)

The study of ideals generated by arbitrary set of minors of a generic matrix has deep roots in commutative algebra, algebraic geometry and combinatorics. In this context, the *polyomino ideal* was introduced in a recent work of A.A. Qureshi as the ideal generated by those sets of 2-minors of a matrix that can be combinatorially characterized as a *polyomino*, that is, a collection of equally sized squares joined edge to edge, similar to a pruned chessboard.

This poster focuses on the theory of the *rook polynomial* of a polyomino and its connection with the corresponding polyomino ideal. The rook problem involves counting the number of ways to place k non-attacking rooks on a given polyomino  $\mathcal{P}$ . The rook number of  $\mathcal{P}$ , denoted by  $r(\mathcal{P})$ , is the maximum number of non-attacking rooks that can be placed it. on

The rook polynomial of  $\mathcal{P}$  is defined as

$$r_{\mathcal{P}}(t) = \sum_{k=0}^{r(\mathcal{P})} r(k, \mathcal{P}) t^k$$

where  $r(k, \mathcal{P})$  is the number of ways to arrange k non-attacking rooks on  $\mathcal{P}$ . For instance, the polyomino  $\mathcal{P}$  here has  $r_{\mathcal{P}}(t) =$  $1 + 11t + 31t^2 + 24t^3$  and  $r(\mathcal{P}) = 3$ .

Determining the rook number and the rook polynomial for a pruned chessboard remains a highly difficult combinatorial

problem. However, recent work has shown that these problems can be approached using algebraic invariants associated with the polyomino ideal. In particular, it has been conjectured that the rook number and the rook polynomial of  $\mathcal{P}$  (and one of its reformulations) correspond, respectively, to the *Castelnuovo–Mumford regularity* and the *h*-polynomial of the coordinate ring of  $\mathcal{P}$ . In this poster, we present the current state of the art, along with several recent results and combinatorial techniques.



### 36. An approach to Martsinkovsky invariant via Auslander approximation theory



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Yuya Otake (Nagoya University, m21012v@math.nagoya-u.ac.jp)

Auslander and Buchweitz [ABu] developed the Cohen–Macaulay approximation theory over Gorenstein local rings, and Auslander, in his unpublished paper [Aus], established the theory of the  $\delta$ -invariant using the Cohen–Macaulay approximation. The delta invariant has been studied by many researchers, and it has interesting connections to ideal theory, as exemplified by index theory [DinI, DinII, Her]. Martsinkovsky [MarI, MarII] extended the theory of the delta invariant to the  $\xi$ -invariant over general Noetherian local rings. We present an approach to analyzing the  $\xi$ -invariant using Auslander's approximation theory, based on the preprint [O].

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## 37. Pseudo-Frobenius numbers and defining ideals in stretched numerical semigroup rings

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This research<sup>4</sup> is based on joint work with Do Van Kien of Hanoi Pedagogical University No. 2 and Naoyuki Matsuoka of Meiji University.

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<sup>&</sup>lt;sup>4</sup>Pseudo-Frobenius numbers and defining ideals in stretched numerical semigroup rings, https://arxiv.org/abs/2501.06415.

Let  $H = \langle a_1, a_2, \ldots, a_n \rangle$  be a numerical semigroup, and R = k[H] be the numerical semigroup ring of H over a field k. Then we denote by  $I_H$  the *defining ideal* of k[H], which is the kernel of the homomorphism  $S = k[X_1, \ldots, X_n] \to k[H]$  with  $X_i \mapsto t^{a_i}$ for  $1 \leq i \leq n$ .

The structure of k[H] is determined by its defining ideal. Therefore, examining the defining ideals, for instance, finding a minimal system of generators, has long been studied as a classical problem in commutative algebra. By J. Herzog's remarkable result<sup>5</sup>, investigating the structure of the defining ideal for 3-generated numerical semigroups has been completely solved. However, the case of 4 or more generators is too complicated to investigate.

Moreover, we have an important invariant

$$PF(H) = \{ z \in \mathbb{Z} \setminus H \mid z+h \in H \text{ for all } h \in H \setminus \{0\} \}$$

and call an element  $\alpha \in PF(H)$  a *pseudo-Frobenius number* of H. Under these notations, we show the following criterion:

**Theorem.** Assume  $k[H]/(t^{a_1})$  is stretched. Then the following are equivalent:

(1) There exist homogeneous elements of positive degree  $f_1, \ldots, f_n, g_1, \ldots, g_n \in S$  such that

$$I_H = I_2 \begin{pmatrix} f_1 & \cdots & f_n \\ g_1 & \cdots & g_n \end{pmatrix}$$

where  $I_2(M)$  denotes the ideal of S generated by 2-minors of a matrix M.

(2) There exist  $h \in \mathbb{Z}_{\geq 0}$  and  $\alpha \in \mathbb{Z}_{\geq 0}$  such that

$$PF(H) = \{h + \alpha, h + 2\alpha, \dots, h + (n-1)\alpha\}.$$

Here, an Artinian local ring  $(A, \mathfrak{m})$  is called *stretched* if the number of minimal generators of  $\mathfrak{m}^2$  is at most 1.

D. T. Cuong, D. V. Kien, N. Matsuoka and H. L. Truong expected that this equivalent holds true in general. This result provides a partial solution of their conjecture.

Thus, these theorem and conjecture are also interesting in that they suggest an extension of Herzog's results to the case of four or more generators.

In this poster, we introduce the main theorem and its applications with examples.

#### 38. Coherent functors, powers of ideals, and asymptotic stability

<sup>&</sup>lt;sup>5</sup>J. Herzog, *Generators and relations of abelian semigroups and semigroup rings*, Manuscripta Mathematica **3** (1970), no. 2, 175-193.

Siddhartha Pramanik (Indian Institute of Technology Kharagpur, siddharthap@kgpian.iitkgp.ac.in, pramaniksiddhartha2@gmail.com)

Let R be a Noetherian ring,  $I_1, \ldots, I_r$  be ideals of R, and  $N \subseteq M$  be finitely generated R-modules. For  $\underline{n} = (n_1, \ldots, n_r) \in \mathbb{N}^r$ , set  $G_{\underline{n}} := M/\mathbb{I}^{\underline{n}}N$ , where  $\mathbb{I}^{\underline{n}} = I_1^{n_1} \cdots I_r^{n_r}$ . Suppose F is a coherent functor on the category of finitely generated Rmodules. We prove that the set  $\operatorname{Ass}_R(F(G_{\underline{n}}))$  of associate primes and  $\operatorname{grade}(J, F(G_{\underline{n}}))$ stabilize for all  $\underline{n} \gg 0$ , where J is a non-zero ideal of R. Furthermore, if the length  $\lambda_R(F(G_{\underline{n}}))$  is finite for all  $\underline{n} \gg 0$ , then there exists a polynomial P in r variables over  $\mathbb{Q}$  such that  $\lambda_R(F(G_{\underline{n}})) = P(\underline{n})$  for all  $\underline{n} \gg 0$ . When R is a local ring, we give a sharp upper bound for the total degree of P. As applications, when R is a local ring, we show that for each fixed  $i \ge 0$ , the *i*-th Betti number  $\beta_i^R(F(G_{\underline{n}}))$  and Bass number  $\mu_R^i(F(G_{\underline{n}}))$  are given by polynomials in  $\underline{n}$  for all  $\underline{n} \gg 0$ . Thus, in particular, the projective dimension  $\operatorname{pd}_R(F(G_{\underline{n}}))$  (resp., injective dimension  $\operatorname{id}_R(F(G_{\underline{n}}))$  is constant for all  $\underline{n} \gg 0$ .

This is joint work with Souvik Dey, Dipankar Ghosh, Tony J. Puthenpurakal, and Samarendra Sahoo.

#### 39. The type of finite complemented affine semigroups

Om Prakash (Chennai Math. Institute, India, omprakash@cmi.ac.in)

We prove that the type of an affine semigroup ring is equal to the number of maximal elements in the Apéry set with respect to the set of exponents corresponding to a maximal regular sequence of monomials. Furthermore, we classify all finite complemented submonoids of  $\mathbb{N}^d$  that have type one.

### 40. Bounds on the Castelnuovo–Mumford regularity and the Ratliff– Rush index

Shruti Priya (Indian Institute of Technology Kharagpur, shruti96312@kgpian.iitkgp.ac.in)

For a Cohen-Macaulay local ring  $(R, \mathfrak{m})$  of dimension d, and I an  $\mathfrak{m}$ -primary ideal, we derive upper bounds on the Ratliff-Rush index in terms of higher Hilbert coefficients and the reduction number of I w.r.t. to a minimal reduction J. In the specific case of two-dimensional Cohen-Macaulay local rings, the established bounds on the Ratliff-Rush index consequently lead to bounds on the Castelnuovo-Mumford regularity of the associated graded ring of I.

#### 41. Stanley–Reisner ideals of higher independence complexes

Amit Roy (Chennai Math. Institute, amitiisermohali493@gmail.com)

For  $t \geq 2$ , the *t*-independence complex  $\operatorname{Ind}_t(G)$  of a graph G is the collection of all  $A \subseteq V(G)$  such that each connected component of the induced subgraph G[A] has at most t-1 vertices. The topology of  $\operatorname{Ind}_t(G)$  is intimately related to the combinatorial property of G. We consider the Stanley–Reisner ideal  $I_t(G)$  of  $\operatorname{Ind}_t(G)$  and focus on their algebraic properties. We prove that for a chordal graph G and for all t

$$\operatorname{reg}(R/I_t(G)) = (t-1)\nu_t(G) \text{ and } \operatorname{pd}(R/I_t(G)) = \operatorname{bight}(I_t(G)),$$

where  $\nu_t(G)$  denotes the induced matching number of the corresponding hypergraph of  $I_t(G)$ , and reg, pd and bight stand for the regularity, projective dimension, and big height, respectively. As a consequence of the above results, we combinatorially characterize when the Stanley–Reisner ideal of *t*-independence complex of a chordal graph has a linear resolution as well as when it satisfies the Cohen–Macaulay property. The above formulas and their consequences can be seen as a nice generalization of the classical results corresponding to the edge ideals of chordal graphs.

# 42. The Ratliff property of the edge ideals of weighted-oriented graphs

Pritam Roy (Indian Institute of Technology Kharagpur, pritamroy.1929@gmail.com)

In this project, we prove that the edge ideal I(D) of each of the following weightedoriented graphs satisfies the Ratliff (or strong persistence) property: graphs having an outward leaf vertex; graphs having an inward leaf vertex, and that leaf vertex has the sink neighbor; graph having an inward leaf vertex, and that leaf vertex has the neighbor of weight 1. Next, we show  $(I(D)^2 : I(D)) = I(D)$  for the edge ideal I(D)of all weighted-oriented graphs and  $(I(D)^3 : I(D)) = I(D)^2$  for each of the edge ideals of weighted-oriented cycles and weighted-oriented trees having an inward leaf vertex. Finally, we prove for a weighted-oriented graph without any source vertex that if a prime ideal (that is not the irrelevant maximal ideal) happens to be an associated prime of some power of the edge ideal, then that is also an associated prime of all the subsequent powers. We also prove for a weighted-oriented graph that has no source vertex but has all vertices of weight more than 1 and has a vertex of in-degree 1, that the irrelevant maximal ideal is an associated prime of every positive power of the edge ideal.

# 43. Numerical characterizations of $S_2$ -ifications of Rees algebras of homogeneous ideals

Sudeshna Roy (Indian Institute of Technology Madras, sudeshnaroy.11@gmail.com)

The multiplicity-based integral dependence criterion goes back to D. Rees and is an essential tool in commutative algebra and singularity theory. However, Rees' theorem works only for ideals that are primary to the maximal ideal in a local ring. Since then, much research has been done to produce such criteria for arbitrary ideals. Recently, with S. Das and V. Trivedi, we obtained new localization-free criteria for integral dependence in terms of well-known invariants. This naturally came up as an application of various density functions that we developed. Our constructions were motivated by the Hilbert-Kunz density function due to Trivedi. When the base ring is a normal domain, one could interpret the above results as numerical characterizations of the graded pieces of the integral closure of the Rees algebra of the ideal in its quotient field. An  $S_2$ -ification criterion of the Rees algebras was first given by C. Ciupercă for ideals primary to the maximal ideal in terms of the first two Hilbert coefficients. He further defined a jmultiplicity type invariant to provide a criterion (involving localization) for arbitrary ideals. This presentation discusses another characterization of the  $S_2$ -ification of the Rees algebra of a homogeneous ideal in a standard-graded normal domain R over a field. For this, we will introduce the beta density function associated with the adic filtration of a homogeneous ideal in R. A novelty of our approach is that it does not involve localization and is amenable to computation.

This poster is based on ongoing joint work with S. Das and H. L. Truong.

#### 44. Toric ideals of weighted oriented graphs

Tapas Kumar Roy (Indian Institute of Technology Kharagpur, tapasroy147@kgpian.iitkgp.ac.in)

We explicitly compute circuit binomials of toric ideals of weighted oriented graphs. A toric ideal is called *strongly robust* if its Graver basis is equal to its set of indispensable binomials. For certain classes of weighted oriented graphs, we prove that its toric ideal is strongly robust. For any weighted oriented graph D, if its toric ideal  $I_D$  is generalized robust or weakly robust, then we show that D has no subgraphs of certain structures. For certain classes of weighted oriented graphs, we prove that robustness, strongly robustness, generalized robustness, weakly robustness are equivalent. We explicitly compute Graver basis of toric ideal of some classes of weighted oriented graphs. If D is a weighted oriented graphs consisting of certain number of cycles sharing a path, then we show that the Graver basis, universal Gröbner basis, reduced Gröbner basis with respect to degree lexicographic order of  $I_D$  coincide. Also, for certain classes of weighted oriented graphs, we show that the Graver basis and the circuit binomials of

its toric ideal are equal.

## 45. Vanishing of co-homologies, and Cohen–Macaulay modules of minimal multiplicity

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Let  $(R, \mathfrak{m}, k)$  be a commutative Noetherian local ring. Let M be a CM (Cohen-Macaulay) R-module of dimension r. It is well-known that the multiplicity  $e(M) \ge \mu(\mathfrak{m}M) + (1-r)\mu(M)$ , where  $\mu(M)$  denotes the minimal number of generators of M. When equality holds, M is said to have minimal multiplicity. For example, a module M of finite length has minimal multiplicity if and only if  $\mathfrak{m}^2 M = 0$ . We show that Cohen-Macaulay modules with minimal multiplicity are Ext-test modules (depending on whether  $e(M) < 2\mu(M)$  or  $e(M) > 2\mu(M)$ ), which detect finiteness of projective and injective dimensions of a given module. Most notably, we verify the long-standing Auslander-Reiten conjecture for every CM module of minimal multiplicity. As consequences of the above results, we show a number of characterizations of various local rings.

### 46. Quasi-pure resolutions and some lower bounds of Hilbert coefficients of Cohen–Macaulay modules

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Let  $(A, \mathfrak{m})$  be a Gorenstein local ring and let M be a finitely generated Cohen-Macaulay A module. Let  $G(A) = \bigoplus_{n\geq 0} \mathfrak{m}^n/\mathfrak{m}^{n+1}$  be the associated graded ring of A and  $G(M) = \bigoplus_{n\geq 0} \mathfrak{m}^n M/\mathfrak{m}^{n+1}M$  be the associated graded module of M. If A is regular and if G(M) has a quasi-pure resolution then we show that G(M) is Cohen-Macaulay. If G(A) is Cohen-Macaulay and if M has finite projective dimension then we give lower bounds on  $e_0(M)$  and  $e_1(M)$ . Finally let  $A = Q/(f_1, \ldots, f_c)$  be a strict complete intersection with  $\operatorname{ord}(f_i) = s$  for all i, and let M be a Cohen-Macaulay module with  $\operatorname{cx}_A(M) = r < c$ , we give lower bounds on  $e_0(M)$  and  $e_1(M)$ .

# 47. Hilbert coefficients versus Buchsbaumness of blow-up algebras

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Let  $(A, \mathfrak{m})$  be a Noetherian local ring of dimension d > 0 with infinite residue field and  $I \subseteq A$  an  $\mathfrak{m}$ -primary ideal with a minimal reduction J. Let  $f_0(I)$  denote the multiplicity of the special fiber cone  $F_{\mathfrak{m}}(I)$ . If A is Cohen-Macaulay and  $f_0(I) = e_1(I) - e_0(I) - e_1(J) + \ell(A/I) + \mu(I) - d + 1$ , then  $F_{\mathfrak{m}}(I)$  need not be Cohen-Macaulay. Let  $\mathcal{I}$  be an I-good filtration. We consider the equality  $e_1(\mathcal{I}) - e_1(J) = 2e_0(\mathcal{I}) - 2\ell(A/I_1) - \ell(I_1/(I_2 + J))$  where  $J \subseteq I$  is a minimal reduction of  $\mathcal{I}$ . In particular for  $\mathcal{I} = \{I^n\}$ , when both the above equalities hold, then we prove, under mild conditions, that (1) if A is generalized Cohen-Macaulay, then  $F_{\mathfrak{m}}(I)$  is generalized Cohen-Macaulay; In addition, if depth A > 0, then depth  $F_{\mathfrak{m}}(I) = \text{depth } A$  and (2) if A is Buchsbaum and depth  $A \geq d - 1$ , then  $F_{\mathfrak{m}}(I)$  is Buchsbaum. We also discuss the Buchsbaumness and I-invariant of the associated graded ring  $G(\mathcal{I})$ . This is collaborative work with Anoot Kumar Yadav.

### 48. Complexity and curvature of pairs of Burch ideals and modules

Mouma Samanta (Indian Institute of Technology Kharagpur, mouma17@kgpian.iitkgp.ac.in)

The complexity and curvature of a module were first introduced by Avramov to distinguish modules of infinite homological dimension. Later, complexity was extended by Avramov–Buchweitz to every pair of modules, which measures the polynomial growth rate of minimal number of generators of their Ext modules. Dao studied a similar notion of Tor-complexity. Recently, Dey–Ghosh–Saha initiated the study of Ext and Tor curvature of a pair of modules, which measures the exponential growth rate of the corresponding Ext and Tor, respectively. On the other hand, the concept of Burch ideals was introduced by Dao–Kobayashi–Takahashi. It includes large well-studied classes of ideals in a Noetherian local ring  $(R, \mathfrak{m}, k)$ . For examples, every non-zero ideal of the form  $\mathfrak{am}$  (e.g.,  $\mathfrak{m}^n$  for  $n \geq 1$ ), and under mild conditions every integrally closed ideal I with depth(R/I) = 0 are Burch ideals. Suppose I and J are Burch ideals such that I is  $\mathfrak{m}$ -primary. One of the main results of this poster is  $\operatorname{cx}_R(I, J) = \operatorname{tcx}_R(I, J) = \operatorname{cx}_R(k)$ . Moreover, we show that R is complete intersection  $\iff \operatorname{cx}_R(I, J) = \operatorname{tcx}_R(I, J) = \operatorname{tc$ 

This is joint work with Dipankar Ghosh.

#### 49. On a relative dependency formula

Shashi Ranjan Sinha (Indian Institute of Technology Hyderabad, ma20resch11005@iith.ac.in)

Celikbas, Liang and Sadeghi established a one-sided inequality for the relative version of Jorgensen's dependency formula and questioned whether it would be an equality. In this poster, we show that the inequality can be indeed strict, and prove a relative dependency formula. Along the way, we obtain some bounds on s(M, N), a notion related to the vanishing of relative homology under specific assumptions.

# 50. Minimal bigraded free resolution of Rees algebra of almost complete intersections

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An almost complete intersection ideal in a polynomial ring can be seen as a d-sequence ideal, with the minimal number of generators being one more than its height. We present the minimal bigraded free resolutions of Rees algebras associated with these ideals, realized as a mapping cone of two complexes. As a result, we can give the explicit bigraded free resolutions of the Rees algebra of almost complete intersections of grade 2 and grade 3. Furthermore, we obtain bounds on the Rees algebra's diagonals (c, e), which guarantee Koszulness and Cohen–Macaulayness of the corresponding diagonal subalgebras. These bounds are given in terms of the invariants associated with the almost complete intersections.

#### 51. Betti numbers of normal edge rings

Zexin Wang (Soochow University, zexinwang6@outlook.com)

We introduce a novel approach named the *induced-subgraph approach* for investigating the Betti numbers of normal edge rings. Utilizing this approach, we compute all the multi-graded Betti numbers of the edge rings associated with two-ear graphs, compact graphs and multi-path graphs. In this context, the *two-ear graph* is a non-bipartite graph introduced by Hibi et al. in 2014, a *compact graph* is a simple graph containing no even cycles while satisfying the odd-cycle condition, and a *multi-path graph* is a simple graph formed by multiple paths with identical start and end vertices. In particular, we show that for two-ear graphs, compact graphs of type 1 or 2 and multi-path graphs, their multi-graded Betti numbers are always equal to the top multi-graded Betti numbers of some of their induced subgraphs. In contrast, some of the multi-graded Betti numbers of compact graphs of type 3 are not the top multi-graded Betti numbers of any of their induced subgraph. We speculate that our approach can be applicable to many other normal edge rings. The poster content is derived from

Z. Wang, D. Lu, The Betti numbers of normal edge rings I, arXiv:2404.10672 and

Z. Wang, D. Lu, The Betti numbers of normal edge rings II, arXiv:2503.03171.

Additionally, this approach can also be applied to compute Betti numbers of certain monomial ideals, as shown in

Z. Wang, D. Lu, *Betti numbers of edge ideals of weighted oriented crown graphs*, arXiv:2505.06992.

## 52. A note on a conjecture of Rossi for reduction numbers of ideals and their Ratliff–Rush filtrations

Anoot Kumar Yadav (Indian Institute of Technology Patna, anoot\_2021ma06@iitp.ac.in)

Let  $(A, \mathfrak{m})$  be a Cohen-Macaulay local ring of dimension  $d \geq 3$  and I an  $\mathfrak{m}$ -primary ideal with a minimal reduction J. In this poster, we prove that  $r_J(I) \leq e_1(I) - e_0(I) + \ell(A/I) + 1$  if  $e_3(I) = e_2(I)(e_2(I) - 1)$  and  $e_i(I) = 0$  for  $i \geq 4$  where  $r_J(I)$  is the reduction number of I with respect to J and  $e_i(I)$  are the Hilbert coefficients. Our result affirms a conjecture of M. E. Rossi. We also prove that (i)  $e_3(\mathcal{I}) \leq e_2(\mathcal{I})(e_2(\mathcal{I}) - 1)$  for any I-admissible filtration  $\mathcal{I}$  and (ii)  $e_3(I) \leq e_2(I)(e_2(I) - e_1(I) + e_0(I) - \ell(A/I))$ for an integrally closed ideal I. To our best knowledge, the above bound for  $e_3(I)$  in the case of  $\mathfrak{m}$ -primary ideals is better than the earlier known bounds. Further, the respective boundary cases in the above bounds along with the vanishing of  $e_i(\mathcal{I})$  for  $4 \leq i \leq d$  force certain "good behaviour of the Ratliff-Rush filtration of  $\mathcal{I}$ " which is a weaker condition than depth  $G_{\mathcal{I}}(A) \geq d - 1$ , however we show that it has many interesting consequences on the Hilbert coefficients. We also discuss bounds for the stability index and the reduction number of the Ratliff-Rush filtration.

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