TITLE AND ABSTRACT

(in the order of the program)

Plenary Talks

Integral Equations with Nonlocal Operators: From Modeling to Computation

Qiang Du (Columbia University, USA)

Recent applications and theoretical advances in integral equation models involving nonlocal operators have demonstrated their potential as effective alternatives to local models, particularly in handling singularities and anomalies. In addition, these nonlocal models often arise as continuum limits of large-scale discrete systems used in data science and network analysis. In this work, we highlight recent progress in the study of such models and discuss related challenges in modeling, analysis, and computation.

A research on numerical computations for high-dimensional stochastic differential equations

Yoshio Komori (Kyushu Institute of Technology, Japan)

We are concerned with numerical methods to calculate weak approximations to solutions for high-dimensional It\^{o} stochastic differential equations (SDEs). As one of the classes of such methods, we have proposed Split S-ROCK methods, which are based on not only orthogonal Runge--Kutta--Chebyshev methods of order 2, but also a Strang splitting-type approach. The methods are of weak order 2 and have high computational accuracy for relatively large time-step size, as well as good stability properties.

Incidentally, as competitors in the point of good stability properties, there is another class of numerical methods. It is the class of stochastic exponential Runge--Kutta methods. A key point for the methods to tackle high-dimensional SDEs is efficient computations related to matrix exponential functions. Some researchers have recently worked on this problem
and proposed some numerical methods. Especially, Caliari and his colleagues have taken it
in the framework of tensor-matrix products, and have provided software packages in MATLAB. In numerical experiments, we will investigate the computational efficiency of these methods for high-dimensional problems.

Invited Talks

Flexible Time Integration Methods for Multiphysics PDE Systems

Daniel R. Reynolds (Southern Methodist University, USA)

Multiphysics models couple two or more physical processes together in a single simulation. These combinations may include systems of differential equations with different type (parabolic, hyperbolic, etc.), with different degrees of nonlinearity, and that evolve on disparate time scales. As a result, such simulations prove challenging for "monolithic" time integration methods that treat all processes using a single approach.

In this talk, I will discuss recent work on time integration methods that allow the flexibility to apply different techniques to distinct physical processes. While such techniques have existed for some time, including additive Runge--Kutta implicit-explicit (ImEx), multirate (a.k.a., multiple time stepping), and operator-splitting methods, there have been comparably few that combine these types of flexibility into a single family, while also supporting high orders of accuracy and temporal adaptivity. In this talk, I focus on newly developed implicit-explicit families of methods for multirate problems, along with novel techniques for time adaptivity in multirate infinitesimal time integration methods.

Fractional-Step Methods: Theory and Practice

Raymond Spiteri (University of Saskatchewan, Canada)

Fractional-step methods are a popular and powerful divide-and-conquer approach for the numerical solution of differential equations. The basic premise is that the right-hand side of the differential equation is split into terms that are integrated separately, often by specialized methods. Fractional-step methods arise from the practical need to solve problems that are beyond the reach of monolithic methods in terms of computational requirements such as memory or runtime.

Although fractional-step methods have been well studied, there is still a lot to learn about their various aspects, in particular, how should one choose the coefficients of the splitting method, the individual sub-integrators, and the order in which they are applied--- not to mention what makes for a good split of the right-hand side in the first place. In this presentation, we offer our experience with some observations in the literature about fractional-step methods in terms of these aspects.

Dynamically regularized Lagrange multiplier method for the Cahn-Hilliard-Navier-Stokes system

Lili Ju (University of South Carolina, USA)

This work is concerned with efficient and accurate numerical schemes for the Cahn-Hilliard-Navier-Stokes phase field model of binary immiscible fluids. By introducing two Lagrange multipliers for each of the Cahn-Hilliard and Navier-Stokes parts, we reformulate the original model problem into an equivalent system that incorporates the energy evolution process. Such nonlinear, coupled system is then discretized in time using first- and second-order backward differentiation formulas, in which all nonlinear terms are treated explicitly and no extra stabilization term is imposed. The proposed dynamically regularized Lagrange multiplier (DRLM) schemes are mass conserving and unconditionally energy stable with respect to the original variables. In addition, the schemes are fully decoupled: each time step involves solving two biharmonic-type equations and two generalized linear Stokes systems, together with two nonlinear algebraic equations for the Lagrange multipliers. A key feature of the DRLM schemes is the introduction of the regularization parameters which ensure the unique determination of the Lagrange multipliers and mitigate the time step size constraint without affecting the accuracy of the numerical solution, especially when the interfacial width is small. Various numerical experiments are presented to illustrate the accuracy and robustness of the proposed DRLM schemes in terms of convergence, mass conservation, and energy stability.

Convergence analysis of the nonoverlapping Robin-Robin method for nonlinear elliptic equations

Eskil Hansen (Lund University, Sweden)

The nonoverlapping Robin-Robin method is commonly encountered when discretizing elliptic equations, as it enables the usage of parallel and distributed hardware. Convergence has been derived in various linear contexts, but little has been proven for nonlinear equations. In this talk we present a convergence analysis for the Robin-Robin method applied to nonlinear elliptic equations with a p-structure, including degenerate diffusion equations governed by the p-Laplacian. The analysis relies on a new theory for nonlinear Steklov-Poincare operators based on the p-structure and the Lp-generalization of the Lions-Magenes spaces. This framework allows the reformulation of the Robin-Robin method into a Peaceman-Rachford splitting on the interfaces of the subdomains, and the convergence analysis then follows by employing elements of the abstract theory for monotone operators. This is joint work with Emil Engström.

On the Equations of Electroporoelasticity

Amnon J Meir (Southern Methodist University, USA)

Complex physical phenomena and systems frequently involve multiple components, complex or coupled domains, complex physics or multi-physics, as well as multiple spacial and temporal scales. Such phenomena are often modeled by systems of coupled partial differential equations, or integro-partial differential equations, often nonlinear. One such phenomenon of interest is electroporoelasticity.

After introducing the equations of electroporoelasticity (the equations of poroelasticity coupled to Maxwell's equations) which naturally arise in geoscience, hydrology, and petroleum exploration, as well as various areas of science and technology, I will describe some recent results (well posedness), the numerical analysis of a finite-element based method for approximating solutions, and some interesting challenges.

Approximation of irregular functionals of SDEs

Dai Taguchi (Kansai University, Japan)

Avikainen (2009) provided a sharp upper bound for the expectation of $|1_{D}(X)-1_{D}(Y)|^{p}$ by the expectation of $|X-Y|^{q}$, for any one-dimensional random variables X with a bounded density function and Y, and intervales D. Then Giles and Xie (2017) give a simple proof. For multi-dimensional case, if both random variables X and Y have bounded densities, then this estimate is generalized by Taguchi, Tanaka and Yuasa (2022) for D with smooth boundary. In this talk, we generalize these results to the case where only one of X and Y has bounded density function and more general domain D (e.g. convex sets). We apply our main result to numerical approximation for irregular functional of a solution to stochastic differential equations (SDEs) based on the Euler--Maruyama scheme and the multilevel Monte Carlo method. This is based on an ongoing research work with Hoang-Long Ngo (Hanoi National University of Education).

Density estimates for jump diffusion processes

Tran Ngoc Khue (Hanoi University of Science &Technology, Vietnam)

We consider a real-valued diffusion process with a linear jump term driven by a Poisson point process and we assume that the jump amplitudes have a centered density with finite moments. We show upper and lower estimates for the density of the solution in the case that the jump amplitudes follow a Gaussian or Laplacian law. The proof of the lower bound uses a general expression for the density of the solution in terms of the convolution of the density of the continuous part and the jump amplitude density. The upper bound uses an upper tail estimate in terms of the jump amplitude distribution and techniques of the Malliavin calculus in order to bound the density by the tails of the solution. We also extend the lower bounds to the multidimensional case. Joint work with Arturo Kohatsu-Higa (Ritsumeikan University) and Eulalia Nualart (Universitat Pompeu Fabra).

Semi-implicit Euler–Maruyama scheme for polynomial diffusions on the unit ball

Takuya Nakagawa (Ritsumeikan University, Japan)

In this lecture, we consider numerical schemes for polynomial diffusions on the unit ball, which are solutions of stochastic differential equations with a diffusion coefficient of the form $\sqrt{1-|x|^{2}}$. We introduce a semi-implicit Euler--Maruyama scheme with the projection onto the unit ball and provide the L^{2}-rate of convergence. The main idea to consider the numerical scheme is the transformation argument introduced by Swart, J. M. for proving the pathwise uniqueness for some stochastic differential equation with a non-Lipschitz diffusion coefficient.

Two-derivative exponential methods for stiff PDEs

Nguyen Van Hoang (Texas Tech University, USA)

For the integration of stiff diffusion-reaction systems, we derive two new classes of exponential methods: two-derivative exponential Runge–Kutta (TDEXPRK) and two-derivative exponential Rosenbrock (TDEXPRB) methods. While TDEXPRK methods are advantageous for problems where stiffness is dominated by the linear part, TDEXPRB methods turn out to be preferable for problems with strong stiffness in both linear and nonlinear parts. Their convergence analysis within the framework of both strongly continuous and analytic semigroups in Banach spaces was performed, leading to the construction of methods up to order 5. Moreover, we present numerical experiments that demonstrate the accuracy and efficiency of the newly derived methods compared to popular classes of exponential integrators for stiff systems, such as exponential Runge–Kutta and exponential Rosenbrock methods.

This work is a collaboration with Dr. Vu Thai Luan and was funded by NSF under award DMS–2309821.

Exponential Nyström methods for systems of second-order differential equations

Pham Quang Huy (Texas Tech University, USA)

Discretizing wave equations in space typically results in large systems of second-order differential equations. For the time integration of such systems, classical Runge-Kutta-Nyström integrators and their extended variants have been widely used. While effective for small or non-stiff problems, these methods often suffer from stability issues and inefficiency when applied to large, stiff or highly oscillatory systems. To overcome these limitations, we develop and analyze a new class of time integration methods, called exponential Nyström (expN) methods. These methods allow significantly larger time steps without sacrificing accuracy. We establish convergence results up to fifth-order accuracy within the framework of strongly continuous semigroups, with error bounds independent of the stiffness or high frequencies of the system. Our numerical experiments demonstrate that the proposed expN methods outperform existing Nyström-type integrators in both efficiency and accuracy.

This work is a collaboration with Prof. V.T. Luan.

A novel efficient approach to the solution of nonlinear boundary value problems

Dang Quang A (Vietnam Academy of Science & Technology, Vietnam)

This paper provides an overview of a novel and efficient approach developed by the author and collaborators for studying both the qualitative behavior and numerical solutions of boundary value problems (BVPs) for high-order nonlinear differential equations. The approach has been recently extended from BVPs with two-point boundary conditions to integral BVPs, and from ordinary differential equations (ODEs) to integro-differential equations, functional differential equations, and partial differential equations. The published results to date demonstrate the efficiency of the proposed approach in comparison with several existing methods. Notably, this methodology is universal and can be applied to a wide range of nonlinear BVPs.

Exact Simulation of Multi-Dimensional Hawkes Processes with Non-Gaussian Intensity

Kazuhiro Yasuda (Hosei University, Japan)

In this talk, we consider an exact simulation of a multivariate Hawkes process. Recently Hawkes processes are often used in finance, modelings of chain-reaction bankruptcies, high-frequency trading and so on. Here, we specially treat multivariate Hawkes process with a non-Gaussian intensity, and provide its exact simulation method which is an extension of an exact simulation algorithm for a single variate Hawkes process with non-Gaussian intensity given in Qu et al. [1]. Finally, we will show some numerical results to illustrate the features of the algorithm. This is a joint work with Toshiki YOSHIDA.

[1] Y. Qu, A. Dassios, and H. Zhao, Efficient simulation of L\'evy-driven point processes, Advances in Applied Probability, 2019, pp.927-966.

Convergence of Markov Chains for Constant Step Size Stochastic Gradient Descent with Separable Functions

David Shirokoff (New Jersey Institute of Technology, USA)

Stochastic gradient descent (SGD) is a popular algorithm for minimizing objective functions that arise in machine learning. Structurally, SGD can be viewed as a random sequence of Euler steps. By viewing the iterates of SGD as a Markov chain, we show that the associated probability laws converge for separable (non-convex) functions and constant step-sizes (learning rates). The characterization answers the question as to which local minima SGD will sample within these settings. We will show that under a standard step-size restriction, the state space decomposes into a transient set and a collection of absorbing sets. The absorbing sets are disjoint closed rectangles, each containing exactly one invariant measure. Examples demonstrating the theory include: a bifurcation where SGD iterates escapes a local minima, and an instance where iterates sample a shallower-narrower well over a deeper-flatter well. We also highlight several ways in which the diffusion approximation, which is often used as an approximation for the SGD probability evolution, fails to capture the long-time dynamics of SGD. Key ingredients in the proof involve viewing the SGD dynamics as a monotone iterated function system, and utilizing results by Dubins and Freedman 1966 and Bhattacharya and Lee 1988, as well as the construction of a Lyapunov function.

Long-term behaviour of symmetric partitioned linear multistep methods

Begoña Cano (University of Valladolid, Spain)

An asymptotic expansion of the global error for partitioned linear multistep methods will provide a tool to analyse the behaviour of these integrators with respect to error growth with time and conservation of invariants. In particular, symmetric partitioned linear multistep methods with no common roots in their first characteristic polynomial, except unity, appear as efficient methods to approximate non-separable Hamiltonian systems since they can be explicit and show good long term behaviour at the same time. As a case study, a thorough analysis will be given for small oscillations of the double pendulum problem, which will be illustrated by numerical experiments.

Innovative Time integrators for Stiff and Highly Oscillatory Problems

Vu Thai Luan (Texas Tech University, USA)

In this talk, we present several classes of innovative time integration methods for stiff and highly oscillatory systems. For first-order stiff evolution problems or parabolic PDEs (e.g., diffusion-reaction systems), we introduce advanced exponential Runge-Kutta/Rosenbrock methods and develop their two-derivative versions. For second-order problems or hyperbolic PDEs (e.g., wave equations), we introduce exponential Nyström integrators. Under reasonable regularity assumptions, we prove convergence results up to fifth-order accuracy for these methods, with error bounds that remain independent of the stiffness or high-frequency components of the problem. Numerical experiments demonstrate the accuracy and efficiency of these new methods compared to existing widely-used schemes in the literature.

The talk includes joint work with my students Hoang Nguyen and Huy Pham, who are supported by the NSF awards DMS–2309821 and DMS–2531805.

Numerical methods for stochastic Volterra integral equations with general singular kernels and jumps

Ngo Hoang Long (Hanoi National University of Education, Vietnam)

We consider a class of stochastic Volterra integral equations with general singular kernels, driven by a Brownian motion and a pure jump Lévy process. We first show that these equations have a unique strong solution under certain regular conditions on their coefficients. Furthermore, the solutions of this equation depend continuously on the initial value and on the kernels. We will then show the regularity of solutions for these equations. Finally, we propose a θ-Euler-Maruyama approximation scheme for these equations and demonstrate its convergence at a certain rate in the L2-norm. Some numerical simulations are also presented to support the theoretical results.
This is a joint work with Phan Thi Huong (Le Quy Don Technique University) and Peter Kloeden (Universität Tübingen)

On the infinite time horizon approximation for Levy-driven McKean-Vlasov SDEs with non-globally Lipschitz continuous and super-linearly growth drift and diffusion coefficients

Luong Duc Trong (Hanoi National University of Education, Vietnam)

This paper studies the numerical approximation for McKean-Vlasov stochastic differential equations driven by Levy processes. We propose a tamed adaptive Euler-Maruyama scheme and consider its strong convergence in both finite and infinite time horizons when applying for some classes of Levy driven McKean-Vlasov stochastic differential equations with non-globally Lipschitz continuous and super linearly growth drift and diffusion coefficients.

Stability and stabilizability of positive switched discrete-time linear singular systems

Do Duc Thuan (Hanoi University of Science &Technology, Vietnam)

In this talk, the positivity, stability and stabilizability of switched discrete-time linear singular (SDLS) systems is studied. Our analysis builds on the recently introduced one-step-map for SDLS systems of index-1 and the stability of positive singular systems. We first provide the positive conditions for SDLS systems and the sufficient stability conditions for positive SDLS systems. After we derive notions and characterizations for stabilizability of positive SDLS systems. Furthermore, we generalize the notion of joint spectral subradius of a finite set of matrix pairs, which allows us to fully characterize stabilizability.

A Mathematical Analysis of Neural Operator Behaviors

Le Vu Anh (Massachusetts Institute of Technology, USA)

Neural operators have emerged as transformative tools for learning mappings between infinite-dimensional function spaces, offering useful applications in solving complex partial differential equations (PDEs). This paper presents a rigorous mathematical framework for analyzing the behaviors of neural operators, with a focus on their stability, convergence, clustering dynamics, universality, and generalization error. By proposing a list of novel theorems, we provide stability bounds in Sobolev spaces and demonstrate clustering in function space via gradient flow interpretation, guiding neural operator design and optimization. Based on these theoretical gurantees, we aim to offer clear and unified guidance in a single setting for the future design of neural operator-based methods.