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# A NON-ARCHIMEDEAN SECOND MAIN THEOREM FOR HYPERSURFACES IN SUBGENERAL POSITION

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Abstract. We apply an idea of Levin to obtain a non-truncated second main theorem for non-Archimedean analytic maps approximating algebraic hypersurfaces in subgeneral position. In some cases, for example when all the hypersurfaces are non-linear and all the intersections are transverse, this improves an inequality of Quang [Thang Long J. Sci. Math. Math. Sci. 2 (2023), pp. 129–143], whose inequality is sharp for the case of hyperplanes in subgeneral position.

Ru [R] observed that in non-Archimedean value distribution theory, the second main theorem without truncation or ramification for non-Archimedean analytic curves approximating algebraic hypersurfaces in projective space follows from the first main theorem. An [A] generalized Ru's approach to projective hypersurfaces in general position with a projective variety. Recently, Quang [Q] applied this approach to hypersurfaces in subgeneral position. Although conjecturally the degree of the hypersurfaces should come into these inequalities, in the works cited above, as with most contemporary work in value distribution theory, only the number of hypersurfaces and their intersection combinatorics enters into the inequalities. Prior work of Levin [L] is one of the few cases where the degrees of the hypersurfaces come into the inequality, although not in what is conjecturally believed to be the optimal way. Levin considered only hypersurfaces in general position. The purpose of this note is to record what Levin's approach yields when the hypersurfaces are allowed to be in subgeneral position. Levin's approach is most useful when the hypersurfaces meet transversely. Although hypersurfaces in subgeneral position do not often meet transversely, when they do, the approach here sometimes gives an improvement on Quang's inequality.

We'll take terminology and notation as in [L, §2].

As in [L], we consider a projective variety $X \subseteq \mathbf{P}^N$ of dimension $n$ over a complete algebraically closed non-Archimedean field $K$ of arbitrary characteristic. We consider $q$ projective hypersurfaces $D_1, \ldots, D_q$ defined over $K$ in $\mathbf{P}^N$. We do not necessarily assume that the $D_j$ are distinct. We assume that $X$ is not entirely contained in any of the hypersurfaces $D_j$. For convenience, we define the dimension

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of the empty set to be $-1$. Then, for each integer $m$ from $-1$ to $n-1$, we define $t_m$ to be the smallest integer such that for every subset $I \subseteq \{1, \dots, q\}$ of cardinality $t_m + 1$, we have

$$\dim\left(\bigcap_{i\in I} D_i \cap X\right) \leq m.$$

Note that if $\dim(X \cap D_1 \cap \cdots \cap D_q) > m$, we take $t_m = q$. Most important for us will be the two numbers $t_{-1}$ and $t_0$. As each hypersurface can lower the dimension by at most one, we have $t_m \geq \min\{n-m, q\}$ for each $m$. When equality holds for all $m$, the hypersurfaces are said to be in general position with $X$. More generally, the hypersurfaces are said to be in $t_{-1}$-subgeneral position with $X$, and this has meaning when $q > t_{-1}$.

Then, with this notation, Quang's subgeneral position without truncation result is:

**Theorem 1** ([Q, Th. 2]). *Let $f : K \to X$ be a non-constant non-Archimedean analytic map not entirely contained in any of the hypersurfaces $D_j$. For $r > 0$,*

$$\sum_{j=1}^{q} \frac{m_f(r, D_j)}{\deg D_j} \leq t_{-1} T_f(r) + O(1).$$

When $X = \mathbf{P}^N$ and the $D_j$ are all hyperplanes $H_j$, then one can easily see that Quang's result is best possible in that the coefficient $t_{-1}$ cannot be reduced.

**Example 2.** Let $H_1, \dots, H_q$ be any collection of not necessarily distinct hyperplanes in $\mathbf{P}^N$. Let $t_{-1}$ be defined as above, namely the smallest integer such that the intersection of any $t_{-1}+1$ of the hyperplanes is empty (or $q$ if no intersection of the hyperplanes is empty). Assume that the absolute value on $K$ is non-trivial, meaning that there is at least one element of $K$ with positive absolute value less than one. Then, there is an algebraically non-degenerate non-Archimedean analytic map $f : K \to \mathbf{P}^N$ such that for all $r > 0$,

$$\sum_{j=1}^{q} m_f(r, H_j) \geq t_{-1} T_f(r) - o(T_f(r)),$$

and so the coefficient $t_{-1}$ in Quang's inequality cannot be reduced.

*Proof.* By the definition of $t_{-1}$, we assume, without loss of generality, that $H_1, \dots, H_{t_{-1}}$ each pass through the point in projective space with coordinates $(0, 0, \dots, 0, 1)$. Let $f_1, \dots, f_N$ be any transcendental entire functions on $K$ such that $T_{f_i}(r) = o(T_{f_j}(r))$ for all $1 \leq i < j \leq N$. For example, since we assumed that the absolute value on $K$ is non-trivial, let $a$ be an element of $K$ such that $0 < |a| < 1$. Then,

$$g(z) = \sum_{n=0}^{\infty} (a^n z)^n$$

has infinite radius of convergence and hence is a transcendental entire function on $K$. Now let $f_0 = 1$, $f_1 = g$, $f_2 = g \circ g$, $f_3 = g \circ g \circ g$, and so on. Then, $T_{f_i}(r) = o(T_{f_j}(r))$ for all $1 \leq i < j \leq N$ by [HY, Th. 2.44]. Now, let $f = (f_0, f_1, , f_2, \dots, f_{N-1}, f_N)$, and observe that $T_f(r) = T_{f_N}(r) + o(T_{f_N}(r))$. Let $P(X_0, \dots, X_N)$ be any non-trivial homogeneous polynomial. Let $k$ be the largest integer such that $X_k$ appears in a monomial in $P$ with non-zero coefficient. If $k = 0$, then trivially, $P \circ f \not\equiv 0$. If

$k > 0$, then let $d$ be the $X_k$-degree of $P$. Then, $T_{P\circ f}(r) = dT_{f_k}(r) + o(T_{f_k}(r))$ by [HY, Th. 2.11], and so $P \circ f \not\equiv 0$, and thus $f$ is algebraically non-degenerate.

Because $H_1, \dots, H_{t_{-1}}$ contain the point $(0, 0, \dots, 0, 1)$, we have that

$$m_f(r, H_j) \geq T_{f_N}(r) - o(T_{f_N}(r)) \qquad \text{for all } j \text{ from 1 to } t_{-1}. \qquad \square$$

*Remark.* If instead, the absolute value on $K$ is trivial, then the only entire functions on $K$ are the polynomials, and all analytic maps from $K$ to $\mathbf{P}^N$ are algebraic. Still, with hyperplanes as in Example 2 and given $\varepsilon > 0$, one easily sees that for all large enough integers $d$, depending on $\varepsilon$,

$$f(z) = (1, z, z^2, \dots, z^{N-1}, z^d)$$

is a linearly non-degenerate map $f : K \to \mathbf{P}^N$ such that for all $r$ sufficiently large,

$$\sum_{j=1}^{q} m_f(r, H_j) \geq (t_{-1} - \varepsilon) T_f(r).$$

One expects to be able to do better when the $D_j$ are non-linear hypersurfaces, but to date, there are few ideas about how to take advantage of higher degree $D_j$ to improve this type of inequality. Levin showed one way to sometimes be able to take advantage of higher degree, particularly when the $D_j$ intersect transversely:

**Theorem 3** ([L, Th. 10]). *Let $X \subseteq \mathbf{P}^N$ be a projective variety over $K$ of dimension $n \geq 1$. Let $D_1, \dots, D_q$ be hypersurfaces in $\mathbf{P}^N$ over $K$ that are in general position with $X$. Let $M$ be the smallest positive integer such that for any subset $I \subseteq \{1, \dots, q\}$ of cardinality $n$,*

$$\bigcap_{i \in I} D_i \cap X \subseteq M \mathrm{Supp}\left(\bigcap_{i \in I} D_i \cap X\right),$$

*where we view $D_1, \dots, D_q$ and $X$ as closed subschemes of $\mathbf{P}^N$. Let $f : K \to X$ be a non-constant non-Archimedean analytic map whose image is not completely contained in any of the hypersurfaces $D_j$. Then, for all $r \geq 1$,*

$$\sum_{j=1}^{q} \frac{m_f(r, D_j)}{\deg D_j} \leq \left(n - 1 + \max_{1 \leq j \leq q} \frac{M}{\deg D_j}\right) T_f(r) + O(1).$$

*Note that if all the intersections amongst $X$ and $D_1, \dots, D_q$ are transverse, then $M = 1$.*

If we relax the general position hypothesis, then Levin's idea gives

**Theorem 4.** *Let $X \subseteq \mathbf{P}^N$ be a projective variety over $K$ of dimension $n \geq 1$. Let $D_1, \dots, D_q$ be not necessarily distinct hypersurfaces in $\mathbf{P}^N$ over $K$. Let $t_0$ and $t_{-1}$ be defined as above. Let $M$ be the smallest positive integer such that for any subset $I \subseteq \{1, \dots, q\}$ of cardinality $t_0 + 1$,*

$$\bigcap_{i \in I} D_i \cap X \subseteq M \mathrm{Supp}\left(\bigcap_{i \in I} D_i \cap X\right),$$

*where we view $D_1, \dots, D_q$ and $X$ as closed subschemes of $\mathbf{P}^N$. Let $f : K \to X$ be a non-constant non-Archimedean analytic map whose image is not completely* *contained in any of the hypersurfaces $D_j$. Let*

$$\alpha = \max_{\substack{I \subseteq \{1,\dots,q\} \\ |I| = t_{-1} - t_0}} \sum_{i \in I} \min\left\{ \frac{M}{\deg D_i}, 1 \right\}.$$

*Then, for all $r \geq 1$,*

$$\sum_{j=1}^{q} \frac{m_f(r, D_j)}{\deg D_j} \leq (t_0 + \alpha) T_f(r) + O(1).$$

Note that if all the intersections amongst $X$ and $D_1, \dots, D_q$ are transverse, then $M = 1$.

*Remark.* In case that the $D_j$ are in general position and $q > n$, then $t_0 = n - 1$, $t_{-1} = n$, $t_{-1} - t_0 = 1$, and

$$\alpha = \max_{1 \leq j \leq q} \min\left\{ \frac{M}{\deg D_j}, 1 \right\},$$

and so we recover Theorem 3. Moreover, $t_0 + \alpha \leq t_{-1}$, and so we also recover Theorem 1.

*Remark.* Here we, as was Levin in [L], are mainly only interested in the case that none of the hypersurfaces $D_i$ are hyperplanes. In Levin's formulation, if any of the $D_i$ are hyperplanes, then Levin's inequality does not improve upon [A]. In a recent extension of Levin's work in a different direction, Huynh [H] shows that Levin's inequality can sometimes be improved in the case where $X = \mathbf{P}^N$, when $q = N$, and when the degeneracy of $f$ is further restricted by not allowing the image of $f$ to be contained in certain hyperplanes tangent to the hypersurfaces $D_i$, which then can give a better result when some of the hypersurfaces are hyperplanes, for example in the case of a non-singular conic and a line meeting transversally in $\mathbf{P}^2$. Combining Huynh's idea with ours might result in some improvement to our inequality under the additional assumption that the image of $f$ is not contained in certain tangent hyperplanes to the hypersurfaces $D_j$, but we do not now see a nice formulation of such a result in a case more general than what Huynh already treats.

Although not typical for hypersurface arrangements not in general position, it is possible for hypersurfaces not in general position to intersect transversely when all the extra intersections happen in dimension zero, for example three conics in $\mathbf{P}^2$ all intersecting transversely in a common set of points. This is precisely the case where Levin's idea gives the best improvement. In the case of three conics intersecting in $\mathbf{P}^2$ transversely in a common set of points, we have $t_0 = 1$ and $t_{-1} = q = 3$, and $\alpha = 1$. In this case, Theorem 1 gives

$$\sum_{j=1}^{3} \frac{m_f(r, D_j)}{2} \leq 3T_f(r) + O(1),$$

which is just the First Main Theorem, but Theorem 4 gives the better

$$\sum_{j=1}^{3} \frac{m_f(r, D_j)}{2} \leq 2T_f(r) + O(1).$$

For some collections of non-linear hypersurfaces, Theorem 4 is a sharp improvement of Theorem 1.

**Example 5.** Let $K$ be any algebraically closed complete non-Archimedean field with characteristic different from three. Let

$$\begin{aligned}
Q_1(X_0, X_1, X_2) &= X_0X_1 - X_2^2,\\
Q_2(X_0, X_1, X_2) &= X_0X_2 - X_1^2,\\
\text{and } Q_3(X_0, X_1, X_2) &= Q_1(X_0, X_1, X_2) + 3Q_2(X_0, X_1, X_2).
\end{aligned}$$

For $j = 1, 2$, and 3, let $D_j$ be the hypersurface in $\mathbf{P}^2$ determined by the vanishing of $Q_j$. Then, the $D_j$ are three conics in $\mathbf{P}^2$ intersecting transversely in the four points $(1, 0, 0)$, $(1, 1, 1)$, $(1, \omega, \omega^2)$, and $(1, \omega^2, \omega)$, where $\omega$ is any primitive third root of unity. Let $f : K \to \mathbf{P}^2$ be the non-constant linear map with coordinate functions $(z, 1, 0)$. Then, for $r \geq 1$,

$$\begin{aligned}
m_f(r, D_1) &= \log \frac{|z|_r^2}{|z|_r} = \log r\\
m_f(r, D_2) &= \log \frac{|z|_r^2}{|-1|_r} = 2\log r\\
\text{and } m_f(r, D_3) &= \log \frac{|z|_r^2}{|z-3|_r} = \log r.
\end{aligned}$$

Hence, for $r \geq 1$,

$$\sum_{j=1}^{3} \frac{m_f(r, D_j)}{2} = 2\log r = 2T_f(r),$$

showing that Theorem 4 is sharp in this case.

*Proof of Theorem* 4. We essentially follow [L, Th. 10] but allow that taking additional hypersurfaces when the intersection has dimension zero may not further reduce the dimension.

Let $f = (f_0, \dots, f_N)$, where $f_0, \dots, f_N$ are entire without common zeros. Let $D_1, \dots, D_q$ be defined by homogeneous polynomials $Q_1, \dots, Q_q$ in $K[X_0, \dots, X_N]$.

If there are fewer than $t_0$ indices $j \in \{1, \dots, q\}$ such that $m_f(r, D_j) \neq O(1)$ as $r \to \infty$, then the theorem follows from the First Main Theorem. We therefore assume from now on that there are at least $t_0$ indices $j$ such that $m_f(r, D_j) \to \infty$ as $r \to \infty$.

Let $I, J \subseteq \{1, \dots, q\}$ be such that $m_f(r, D_i) = O(1)$ for all $i \in I$, and such that $m_f(r, D_j) \to \infty$ as $r \to \infty$ for all $j \in J$. Let $r_0$ be large enough so that for all $i \in I$, all $j \in J$, and all $r \geq r_0$,

$$\frac{m_f(r, D_j)}{\deg D_j} \geq \frac{m_f(r, D_i)}{\deg D_i}.$$

Fix $r \geq \max\{1, r_0\}$. After reindexing, we may assume that

$$\frac{m_f(r, D_1)}{\deg D_1} \geq \frac{m_f(r, D_2)}{\deg D_2} \geq \cdots \geq \frac{m_f(r, D_q)}{\deg D_q}.$$

If $t_{-1} < q$, then

$$D_1 \cap \cdots \cap D_{t_{-1}} \cap D_{t_{-1}+1} = \emptyset.$$

Hilbert's Nullstellensatz then tells us that for each of the coordinate functions $X_j$, there is some power $m_j$ and some homogeneous polynomials $A_{ji}$ such that

$$X_j^{m_j} = \sum_{i=1}^{t_{-1}+1} A_{ji} Q_i.$$

Of course, $\deg A_{ji} = m_j - \deg D_i$. Thus, there exists a constant $C_1$, depending only on the polynomials $A_{ji}$, such that for all $j = 0, \ldots, N$,

$$|f_j|_r^{m_j} \leq C_1 \max_{1 \leq j \leq t_{-1}+1} |f|_r^{m_j - \deg D_i} |Q_i \circ f|_r.$$

Choosing $j$ so that $|f_j|_r = |f|_r$ and canceling $|f|_r^{m_j}$ from both sides then gives

$$1 \leq C_1 \max_{1 \leq i \leq t_{-1}+1} \frac{|Q_i \circ f|_r}{|f|_r^{\deg D_i}} \leq C_1 \frac{|Q_{t_{-1}+1} \circ f|_r}{|f|_r^{\deg D_{t_{-1}+1}}}.$$

Hence,

$$\frac{m_f(r, D_i)}{\deg D_i} \leq \frac{\log C_1}{\deg D_i}, \text{ for all } i > t_{-1}. \tag{1}$$

Note that the constant $C_1$ was chosen depending on the $A_{ji}$, which depend on $r$ in the sense that they depend on how we indexed $Q_1, \ldots, Q_q$. As there are only finitely many ways of reindexing, $C_1$ can be chosen independent of $r$.

If $t_0 = t_{-1} = q$, then the theorem follows from the First Main Theorem, or from Theorem 1, so we henceforth assume that $t_0 < q$. Levin's idea allows us to handle $m_f(r, D_j)$ for $j = t_0 + 1, \ldots, t_{-1}$. Fix one such index $j$. By the definition of $t_0$,

$$X \cap D_1 \cap \cdots \cap D_{t_0} \cap D_j$$

is a finite set of points $\{P_1, \ldots, P_s\}$. We now want to choose hyperplanes $H_1, \ldots, H_s$ so that the following conditions are satisfied:

- $P_i \in H_i$ for $i = 1, \ldots, s$;
- $P_j \notin H_i$ for all $j \neq i \in \{1, \ldots, s\}$;
- If we define $E_i = D_i$ for $i = 1, \ldots, t_0$ and $E_i = H_{i-t_0}$ for $i = t_0+1, \ldots, t_0+s$, then for any index set $I \subseteq \{1, \ldots, t_0 + s\}$ with cardinality $|I| \geq t_0 + 2$,
$$X \cap \bigcap_{i \in I} E_i = \emptyset;$$
- The image of $f$ is not completely contained in any of the $H_i$.

We now re-order the $E_i$ so that

$$\frac{m_f(r, E_1)}{\deg E_1} \geq \cdots \geq \frac{m_f(r, E_{t_0+s})}{\deg E_{t_0+s}}.$$

By our choice of the hyperplanes, if $s \geq 2$, then

$$X \cap E_1 \cap \cdots \cap E_{t_0} \cap E_{t_0+1} \cap E_{t_0+2} = \emptyset.$$

As before, we apply the Nullstellensatz, to find a constant $C_2$ such that for all $i > t_0 + 1$,

$$\frac{m_f(r, E_i)}{\deg E_i} \leq \frac{\log C_2}{\deg E_i}, \tag{2}$$

where $C_2$ appears to depend on $r$ and our choice of the index $j$, as it depends on $\{P_1, \ldots, P_s\}$ and the choice of hyperplanes $H_1, \ldots, H_s$, and these depend on the index $j$ and on our original re-indexing of $D_1, \ldots, D_q$. But again, there are only finitely many ways to re-index $D_1, \ldots, D_q$, and for each reindexing, there are only finitely many choices of the index $j$, and so $C_2$ can be taken to be independent of $r$.

By enlarging $r_0$ if necessary and by our assumption that there were at least $t_0$ indices $j$ such that $m_f(r, D_j) \to \infty$ as $r \to \infty$, we conclude that all the $E_i$ in (2) are, in fact, hyperplanes. Hence,

$$\sum_{i=1}^{s} m_f(H_i, r) \le T_f(r) + O(1). \tag{3}$$

Now, choose linear defining forms $L_i$ for each of the hyperplanes $H_i$. By our definition of $M$,

$$X \cap D_1 \cap \cdots \cap D_{t_0} \cap D_j \subseteq M\mathrm{Supp}\left(X \cap D_1 \cap \cdots \cap D_{t_0} \cap D_j\right).$$

Thus, there exist homogeneous polynomials $B_1, \ldots, B_{t_0}$ and $B_j$ with

$$\deg B_i = Ms - \deg D_i$$

such that

$$(L_1 \cdots L_s)^M - B_j Q_j - \sum_{i=1}^{t_0} B_i Q_i$$

vanishes on $X$. Therefore, there is a constant $C_3$, depending only on the polynomials $B_i$, and hence can be taken to be independent of $r$ if we consider all possible reindexings of $Q_1, \ldots, Q_q$, such that

$$\prod_{i=1}^{s} |L_i \circ f|_r^M \le C_3 \max_{k \in \{1,\ldots,t_0\} \cup \{j\}} |f|_r^{Ms - \deg D_k} |Q_k \circ f|_r \le C_3 |f|_r^{Ms} \frac{|Q_j \circ f|_r}{|f|_r^{\deg D_j}}.$$

Dividing both sides by $|f|_r^{Ms}$ and taking logarithms, we get

$$m_f(r, D_j) \le M \sum_{i=1}^{s} m_f(r, H_i) + C_4,$$

with $C_4$ independent of $r$, again by observing that there are only finitely many ways to reindex the $D_j$. Combining with (3), we conclude

$$\frac{m_f(r, D_j)}{\deg D_j} \le \frac{M}{\deg D_j} T_f(r) + O(1).$$

Note that, in any case,

$$\frac{m_f(r, D_j)}{\deg D_j} \le T_f(r) + O(1)$$

by the First Main Theorem, and so we in fact have

$$\frac{m_f(r, D_j)}{\deg D_j} \le \min\left\{\frac{M}{\deg D_j}, 1\right\} T_f(r) + O(1). \tag{4}$$

We now split the left-hand-side of the inequality we want to prove into three pieces:

$$\sum_{j=1}^{q} \frac{m_f(r, D_j)}{\deg D_j} \le \sum_{j=1}^{t_0} \frac{m_f(r, D_j)}{\deg D_j} + \sum_{j=t_0+1}^{t_{-1}} \frac{m_f(r, D_j)}{\deg D_j} + \sum_{j=t_{-1}+1}^{q} \frac{m_f(r, D_j)}{\deg D_j}.$$

Using (1), we can replace the last piece with $O(1)$. Now, by our choice of ordering for the $D_j$ and (4), we can reduce the middle piece to

$$\sum_{j=t_0+1}^{t_{-1}} \frac{m_f(r, D_j)}{\deg D_j} \leq \alpha T_f(r) + O(1),$$

where

$$\alpha = \max_{\substack{I \subseteq \{1, \ldots, q\} \\ |I| = t_{-1} - t_0}} \sum_{i \in I} \min \left\{ \frac{M}{\deg D_i}, 1 \right\}.$$

Putting those together and applying the First Main Theorem one final time, we now have

$$\begin{aligned} \sum_{j=1}^{q} \frac{m_f(r, D_j)}{\deg D_j} &\leq \sum_{j=1}^{t_0} \frac{m_f(r, D_j)}{\deg D_j} + \alpha T_f(r) + O(1) \\ &\leq t_0 T_f(r) + \alpha T_f(r) + O(1) \\ &\leq (t_0 + \alpha) T_f(r) + O(1). \end{aligned}$$

□

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