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State estimation for nonlinear systems using a recurrent neural network learning algorithm and an event-triggered state observer

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Abstract In this paper, we propose a novel method to estimate the states of nonlinear systems. A recurrent neural network learning algorithm is first developed to predict the nonlinear systems. Then, an event-triggered state observer is designed for the recurrent neural network. This state observer robustly estimates state vTariables of the nonlinear systems. A sufficient condition in terms of a convex optimization problem for the existence of the event-triggered state observer is established. In contrast with the abundance of state estimation methods based on time-triggered state observers where the measurements are always continuously available, the ones in this paper are updated when an event-triggered condition holds. Therefore, it lessens the stress on communication resources while still maintaining an estimation performance. The obtained theoretical analysis is applied to estimate the electrical angular velocity, the electrical angle, and the currents of the permanent magnet synchronous motor.

1 Introduction

The operation of many engineering systems such as state feedback control systems, system supervision, fault diagnosis of dynamic systems, and general diagnosis issues are based on available information on state vectors (see, for example, [1, 5, 9, 19, 22]). However, due to technical or economic reasons, people usually use information state vector estimation instead of measuring the correct one. A typical example of this statement is the permanent magnet synchronous motors (PMSMs) [2, 25], which are brushless drives with all the properties required for servo applications [17]. In the PMSMs, the phase current must be a sinusoidal function of the rotor position. A high-resolution sensor is needed to obtain position information with appropriate resolution. Speed information may be derived from the position sensor or measured by a tachometer. These mechanical sensors increase the shaft inertia and dynamic friction, adding to the cost of the drive. They also need extra wiring beyond the cables required for supplying proper currents to the motor windings. These connections between the motor and the control system are often the source of an overall decrease in reliability. In order to reduce their cost and increase their sobriety and reliability, PMSMs are not always equipped with mechanical sensors (rotor position and velocity). Instead, state observers are proposed to provide state variables of the PMSMs. This approach is very significant since electrical sensors tend to be cheaper and easier to maintain than mechanical ones.

Many methods are proposed in the literature to solve the problem of estimating the state vector of the PMSMs [2, 3, 6, 17, 23, 25]. In particular, nonlinear full-order observers are discussed in [13, 16, 20], while an extended Kalman filter is implemented in [2, 6] to estimate speed and rotor position. However, the above methods [2, 6, 13, 16, 20] did not consider the issue of the unknown load torque, which may lead to large estimation errors. To overcome this limitation, the authors of the work [23] proposed a nonlinear extended observer to estimate the state vector of a PMSM subject to an unknown load torque. Recently, there have been some interesting methods solving

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the state estimation problem of the PMSMs, for example, improved square root UKF [25], a nonlinear Luenberger approach for a non-observable system [3], sliding mode observer [27].

It is worth noting that all existing methods for estimating state vector of the PMSMs [2, 3, 6, 13, 16, 20, 23, 25, 27], were implemented with an assumption that the output vectors are continuous, which may not save communication resources in practical applications. So far, the methods in [2, 3, 6, 13, 16, 20, 23, 25, 27], has not been extended to event-triggered state estimation [8, 9, 11, 12, 15], which is useful in saving communication resource.

Recurrent neural networks (RNNs) have been used as a power technique to solve several practical problems (see, for example, [10, 14, 24, 26]). In particular, a RNN learning algorithm is proposed in [10] to estimate the states of a PMSM, while the high-order neural network structures were studied in [14]. The design of model predictive control systems by using a RNN was reported in [24]. In [26], the exponential stability problem was considered for uncertain stochastic Hopfield neural networks. It is worth noting that the structure of RNNs is more advantageous in designing event-triggered state observers than other nonlinear dynamical systems. Therefore, our main aim in this paper is to develop the RNN learning algorithm in [10, 14, 24] to design event-triggered state observers for the nonlinear systems. The main contributions of this paper are: (1) A RNN model is trained to estimate the states of the nonlinear systems; (2) A new event-triggered state observer is designed to estimate the state vectors of the obtained RNN model; (3) an existence condition of such observer in terms of LMIs is established; and (4) numerical results of the PMSM are provided to demonstrate the applicability of the proposed method.

Notation: A^T is the transpose of A. $||\cdot||$ denotes the Euclidean norm. $\mathbb{R}^n$ is the $n-$ dimensional linear vector space over $\mathbb{R}$. $P > 0$ means that $x^T Px > 0$, $\forall x \neq 0$.

2 Preliminaries and problem statement

Let us consider the following nonlinear systems:

$$\dot{x}(t) = f(x(t), u(t), d(t)), \ \ t \geq 0, \tag{1}$$

$$x(0) = \phi(0), \tag{2}$$

$$y(t) = Cx(t), \tag{3}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^p$ is the output vector, $d(t) \in \mathbb{R}^\ell$ is the external disturbance vector, $f(.)$ is a matrix function, $C \in \mathbb{R}^{p\times n}$ is a constant matrix, $\phi(\theta)$ is a continuous initial function.

The following assumptions are used to obtained the main results of the paper:

(A_1) The system states of the system (1) operate in a bounded region $\mathcal{R}$;
(A_2) There exists a positive constant $\bar{d}$ such that: $||d(t)|| \leq \bar{d}, \ \ \forall t \geq 0$;
(A_3) $||f(x, u, d) - f(z, u, 0)|| \leq \tau_1 ||x - z|| + \tau_2 \bar{d}$, $\forall x, z \in \mathcal{R}$, and $u(t)$ is norm bounded, τ_1, τ_2 are positive scalars.

3 Main result

3.1 Approximating the nonlinear system by a recurrent neural networks

The nonlinear system (1) is predicted by the following RNN:

$$\dot{z}(t) = f_{rnn}(z(t), u(t)) = Az(t) + \begin{bmatrix} \Omega_z \ \Omega_u \end{bmatrix} \begin{bmatrix} \eta(z(t)) \\ u(t) \end{bmatrix}, \ \ t \geq 0, \tag{4}$$

$$z(0) = \phi(0), \tag{5}$$

$$\tilde{y}(t) = Cz(t), \tag{6}$$

where $z(t) \in \mathbb{R}^n$, $\eta(z(t)) = \begin{bmatrix} \eta(z_1(t)) \\ \vdots \\ \eta(z_n(t)) \end{bmatrix}$, $u(t) = \begin{bmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{bmatrix}$, $\eta(\cdot)$ is the activation function satisfying the following inequality

$$|\eta_i(v_1) - \eta_i(v_2)| \leq \bar{\eta}_i |v_1 - v_2|, \ \forall v_1, v_2 \in \mathbb{R}, \tag{7}$$

where $\bar{\eta}_i > 0$ for $i = 1, 2, \ldots, n$ are positive scalars.

$A = -\text{ diag }\{a_1, a_2, \ldots, a_n\}$, $\begin{bmatrix} \Omega_z \\ \Omega_u \end{bmatrix} = \Omega$, $\Omega = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_n \end{bmatrix} \in \mathbb{R}^{(n+m)\times n}$ $(\theta_i = b_i \begin{bmatrix} w_{i1} \\ \vdots \\ w_{i(n+m)} \end{bmatrix}^T$, $a_i > 0$, b_i are constants, for $i = 1, \ldots, n$, $j = 1, \ldots, n+m$, matrices w_{ij} are the weight connecting from the *jth* input to the *ith* neuron, which will be optimized during training. Optimal weighs a_i^* and θ_i^* for (4) are determined as below:

$$(a_i^*, \theta_i^*) = \arg\min_{a_i, \theta_i} \frac{1}{2} \left\| \begin{bmatrix} [\Theta_1]_i & \bar{y}_1^\top \\ [\Theta_2]_i & \bar{y}_2^\top \\ \vdots & \\ [\Theta_N]_i & \bar{y}_N^\top \end{bmatrix} \begin{bmatrix} a_i \\ \theta_i \end{bmatrix} - \begin{bmatrix} f_i(s_1, u_1) \\ f_i(s_2, u_2) \\ \vdots \\ f_i(s_N, u_N) \end{bmatrix} \right\|^2. \tag{8}$$

Remark 1 Different from the RNN in [10], which is only used to predict a particular nonlinear model (namely, the nonlinear function is $f(x(t), u(t), d(t)) = \begin{bmatrix} \frac{3n_p^2(L_d - L_q)}{2J} x_4 x_3 - \frac{n_p}{J} s_L \\ 0 \\ \frac{L_d}{L_q} x_1 x_4 \\ \frac{L_q}{L_d} x_1 x_3 \end{bmatrix}$), the one in this paper can be used to predict a general nonlinear model. Therefore, the RNN reported in [10] can be regarded as a special case of the RNN in this paper.

The following lemma indicates that the error between the state of the nonlinear system and the recurrent neural networks is bounded.

Lemma 1 Let assumptions (A_1), (A_2), and (A_3) be satisfied. If $||f(z(t), u(t), 0) - f_{rnn}(z(t), u(t))|| \leq \bar{\omega}$, then the following inequality holds:

$$||e(t)|| \leq \frac{\tau_2 \bar{d} + \bar{\omega}}{\tau_1}(e^{\tau_1 t} - 1), \ \ t > 0, \tag{9}$$

where $e(t) = x(t) - z(t)$ and $\bar{\omega}$ is a positive number.

Proof We have

$$\begin{aligned} \frac{d}{dt}||e(t)|| \leq ||\dot{e}(t)|| &= ||\dot{x}(t) - \dot{z}(t)|| = ||f(x(t), u(t), d(t)) - f_{rnn}(z(t), u(t))|| \\ &= ||f(x(t), u(t), d(t)) - f(z(t), u(t), 0) \\ &\quad + f(z(t), u(t), 0) - f_{rnn}(z(t), u(t))||. \end{aligned} \tag{10}$$

On the other hand, for all $x, z \in \mathcal{R}$, the following inequality is satisfied:

$$||f(x(t), u(t), d(t)) - f(z(t), u(t), 0)|| \leq \tau_1 ||x - z|| + \tau_2 \bar{d}. \tag{11}$$

By using (36), (11) and inequality $||f(z(t), u(t), 0) - f_{rnn}(z(t), u(t))|| \leq \bar{\omega}$, we obtain

$$\frac{d}{dt}||e(t)|| \leq \tau_1 ||e(t)|| + \tau_2 \bar{d} + \bar{\omega}. \tag{12}$$

Since τ_1 and $\tau_2 \bar{d} + \bar{\omega}$ are positive numbers, under the zero initial condition, the inequality (9) is obtained. The proof is completed. □

3.2 State estimation for the recurrent neural networks by using an event-triggered state observer

In the following, we extend the results reported in [10] to estimate the states of the RNN (4). For this, the following event-triggered state observer is proposed

$$\dot{\hat{z}}(t) = A\hat{z}(t) + \Omega_z\eta(\hat{z}(t)) + \Omega_u u(t) + K(\overline{y}(s_k) - C\hat{z}(s_k)),\ \ t \in [s_k,\, s_{k+1}), \tag{13}$$

where the observer gain matrix will be designed, and the triggering instants $\{s_k\}_{k\in\mathbb{N}}$ is determined by the following ETM:

$$s_0 = 0,\ \ s_{k+1} = s_k + h\min\Big\{\xi \in \mathbb{N}^+ \mid \mathcal{H}(e_\varepsilon(t),\, \varepsilon(s_k)) > \gamma(s_k),\ \ h > 0\Big\}, \tag{14}$$

where $\mathcal{H}(e_\varepsilon(t),\, \varepsilon(s_k)) = \alpha[e_\varepsilon(t)^T(t)\Xi e_\varepsilon(t) - \mu\varepsilon^T(s_k)\Gamma\varepsilon(s_k)]$, $\varepsilon(t) = z(t) - \hat{z}(t)$, $e_\varepsilon(t) = \varepsilon(s_k) - \varepsilon(s_k + \xi h)$, $\xi \in \mathbb{N}$, $\alpha,\, \mu \in (0,\, \infty)$, $\Xi > 0$, and

$$\dot{\gamma}(t) = -\zeta\gamma(t) + \mu\varepsilon^T(s_k)\Gamma\varepsilon(s_k) - e_\varepsilon^T(t)\Gamma e_\varepsilon(t),\ \ \zeta \in (0,\, \infty),\ \ \gamma(0) = 0. \tag{15}$$

By denoting $\eta_{z\hat{z}}(t) = \eta(z(t)) - \eta(\hat{z}(t))$ and $\tau(t) = t - s_k - rh$, $t \in I_r$, the following error dynamic system is obtained:

$$\dot{\varepsilon}(t) = A\varepsilon(t) + \Omega_z\eta_{z\hat{z}}(t) - KC\varepsilon(t - \tau(t)) - KCe_\varepsilon(t),\, t \in [s_k + \sigma_k,\, s_{k+1} + \sigma_{k+1}), \tag{16}$$

$$\varepsilon(s) = \varepsilon(0),\ \ s \in [-h,\, 0]. \tag{17}$$

The following theorem can be considered as the general case of Theorem 3.1 in [10]. It allows us to determine the gain matrix K such that the error dynamic system (16) is asymptotically stable (Figs. 1, 2).

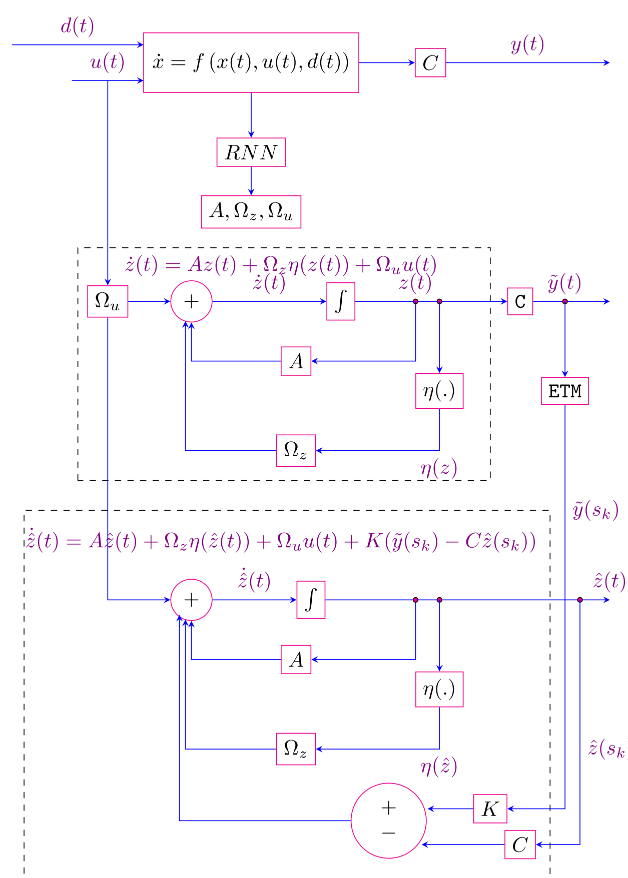

Fig. 1 Schematic of the event-triggered state observer (13)

Theorem 1 Given $\theta > 0$, system (16) is asymptotically stable if there exist $\mathtt{P} > 0$, $\mathtt{Q} > 0$, $\mathtt{R} > 0$, $\Gamma > 0$, Z, X, non-singular S, $\delta \in (0, \infty)$, such that for $\beta \in \{0, 1\}$:

$$\Phi^\star(\beta) = \begin{bmatrix} \Phi(\beta) & \nabla \\ * & -\delta I_n \end{bmatrix} < 0, \tag{18}$$

$$\Phi = \begin{bmatrix} \text{diag }(\mathtt{R},\, \mathtt{R}) & Z \\ * & \text{diag }(\mathtt{R},\, \mathtt{R}) \end{bmatrix} > 0, \tag{19}$$

where

$$\begin{aligned}
\Phi(\beta) &= \Phi_1(\beta) + \Phi_2(\beta) + \delta\bar{\eta}_{\max}^2 \nu_1^T \nu_1,\ \bar{\eta}_{\max} \\
&= \max\{\bar{\eta}_1, \ldots, \bar{\eta}_n\}, \\
\Phi_1(\beta) &= \text{ sym } \{\Psi_\beta^T P \Gamma\} + \nu_1^T \mathtt{Q} \nu_1 - \nu_3^T \mathtt{Q} \nu_3 + h^2 \nu_4^T R_2 \nu_4 - \Delta^T \Phi \Delta \\
&\quad + \mu\nu_2^T \Gamma \nu_2 + \mu\nu_7^T \Gamma \nu_7 - \text{ sym } \Big\{ \big[\, \nu_1^T\ \nu_4^T \,\big] \\
&\quad \begin{bmatrix} \theta S \\ S \end{bmatrix} \nu_4 \Big\},\ \Phi_2(\beta) = \text{ sym } \Big\{ \big[\, \nu_1^T\ \nu_4^T \,\big] \begin{bmatrix} \theta S \\ S \end{bmatrix} A\nu_1 - \big[\, \nu_1^T\ \nu_4^T \,\big] \\
&\quad \begin{bmatrix} \theta X \\ X \end{bmatrix} C\nu_2 - \big[\, \nu_1^T\ \nu_4^T \,\big] \begin{bmatrix} \theta X \\ X \end{bmatrix} C\nu_7 \Big\}, \\
&\Psi_\beta = \big[\, \nu_1^T\ \beta h \nu_5^T + (1-\beta) h \nu_6^T \,\big]^T,\ \Gamma = \big[\, \nu_4^T\ (\nu_1^T - \nu_3^T) \,\big]^T, \\
&\Delta = \big[\, \Delta_1\ \Delta_2\ \Delta_3\ \Delta_4 \,\big]^T,\ \Delta_1 = \big[\, (\nu_1 - \nu_2)^T \,\big], \\
&\Delta_2 = \big[\, \sqrt{3}(\nu_1 + \nu_2 - 2\nu_5)^T \,\big],\ \Delta_3 = \big[\, (\nu_2 - \nu_3)^T \,\big], \\
&\quad \Delta_4 = \big[\, \sqrt{3}(\nu_2 + \nu_3 - 2\nu_6)^T \,\big], \\
&\nabla = \big[\, \nu_1^T\ \nu_4^T \,\big] \\
&\quad \begin{bmatrix} \theta S \\ S \end{bmatrix} \times \Omega_z \big[\, I_n\ 0_{n\times n} \,\big], \\
&\nu_i = \big[\, 0_{n\times(i-1)n}\ I_n\ 0_{n\times(6-i)n}\ 0_{n\times n} \,\big] \in \mathbb{R}^{n\times(7n+n)},\ i = 1, \ldots, 6, \\
&\nu_7 = \big[\, 0_{n\times 6n}\ I_n \,\big] \in \mathbb{R}^{n\times(6n+n)}.
\end{aligned}$$

The observer gain matrix K is obtained as

$$K = S^{-1} X. \tag{20}$$

Proof We denote $\tilde{e}(t) = \left[\varepsilon^T(t) \int_{t-h}^t \varepsilon^T(s) ds \right]^T$ and consider the following Lyapunov function:

$$\begin{aligned}
V(t) = \gamma(t) + \tilde{e}^T(t) \mathtt{P} \tilde{e}(t) + \int_{t-h}^t \varepsilon^T(s) \mathtt{Q} \varepsilon(s) ds \\
+ h \int_{-h}^0 \int_{t+\xi}^t \dot{\varepsilon}^T(s) \mathtt{R} \dot{\varepsilon}(s) ds.
\end{aligned} \tag{21}$$

In light of the proof of Lemma 4 in [9], it is proved that $\gamma(t) \geq 0$ and thus $V(t) \geq 0,\ \forall t > 0$. We have the following estimate:

$$\begin{aligned}
\dot{V}(t) = &- \lambda\gamma(t) + \mu(\varepsilon(t - \tau(t)) + \nu_\varepsilon(t))^T \Gamma (\varepsilon(t - \tau(t)) + \nu_\varepsilon(t)) + \\
&+ 2\zeta^T(t) \Psi_\beta^T P \Gamma \zeta(t) + \zeta^T(t) [\nu_1^T \mathtt{Q} \nu_1 - \nu_3^T \mathtt{Q} \nu_3] \zeta(t) + \\
&+ h^2 \zeta^T(t) (\nu_4^T \mathtt{R} \nu_4) \zeta(t) - h \int_{t-\tau(t)}^t \dot{\varepsilon}^T(s) \mathtt{R} \dot{\varepsilon}(s) ds \\
&- h \int_{t-h}^{t-\tau(t)} \dot{\varepsilon}^T(s) \mathtt{R} \dot{\varepsilon}(s) ds,
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
\zeta(t) &= \left[\zeta_1(t)\ \zeta_2(t)\ \zeta_3(t)\ \zeta_4(t)\right]^T,\\
\zeta_1(t) &= \left[\varepsilon^T(t)\ \varepsilon^T(t-\tau(t))\right],\\
\zeta_2(t) &= \left[\varepsilon^T(t-h)\ \dot{\varepsilon}^T(t)\right],\\
\zeta_3(t) &= \left[\ \tfrac{1}{\tau(t)}\int_{t-\tau(t)}^{t}\varepsilon^T(s)ds\right],\\
\zeta_4(t) &= \left[\tfrac{1}{h-\tau(t)}\int_{t-h}^{t-\tau(t)}\varepsilon^T(s)ds\ e_\varepsilon^T(t)\right].
\end{aligned} \tag{23}$$

Now, by employing the Wirtinger-based integral inequality [21], the reciprocally convex combination inequality [18], the Cauchy matrix inequality and the Schur Complement Lemma [4], one gets

$$\dot{V}(t) \le \zeta^T(t)\Phi^\star(\beta)\zeta(t), \tag{24}$$

where $\beta \in (0,\,1)$.
Thus,

$$\dot{V}(t) < 0 \tag{25}$$

holds if $\Phi > 0$ and $\Phi(\beta) < 0,\ \ \forall\beta \in (0,\,1)$. Since $\Phi^\star(\beta)$ is convex with respective to β, $\Phi^\star(\beta) < 0\ \forall\beta \in \{0,\,1\}$ implies $\Phi^\star(\beta) < 0\ \forall\beta \in (0,\,1)$. Therefore, (16) is asymptotically stable. The proof is completed. □

Remark 2 Provided that $e^{\sigma\xi h} < \alpha\sigma + 1$ holds, where $\mu,\,\sigma,\,\alpha$ are positive scalars and ξ is the smallest integer number satisfying $h \le s_{k+1} - s_k \le \xi h$. For $\gamma(t)$ defined in (15), we have $\gamma(t) \ge 0$ for all $t > 0$. Indeed, for all $t \in [s_k,\,s_{k+1})$, (14) indicates that

$$-e_\varepsilon(t)^T(t)\Xi e_\varepsilon(t) \ge -\frac{1}{\alpha}\gamma(s_k). \tag{26}$$

It follows from (15) and (26) that

$$\gamma(t) \ge \Big(e^{-\sigma(t-s_k)}(1+\frac{1}{\alpha\sigma}) - \frac{1}{\alpha\sigma}\Big)\gamma(s_k) \ge \Big(e^{-\sigma\xi h}(1+\frac{1}{\alpha\sigma}) - \frac{1}{\alpha\sigma}\Big)\gamma(s_k). \tag{27}$$

From inequalities (15), (26), (27) and the assumption $e^{\sigma\xi h} < \alpha\sigma + 1$, the following inequalities satisfied:

$$\begin{aligned}
\frac{d}{dt}\gamma(t) &\ge -\sigma\gamma(t) - \frac{1}{\alpha}\gamma(s_k)\\
&\ge -\Big(\sigma + \frac{1}{\alpha(e^{-\sigma\xi h}(1+\frac{1}{\alpha\sigma}) - \frac{1}{\alpha\sigma})}\Big)\gamma(t),
\end{aligned} \tag{28}$$

which indicates that $\gamma(t) \ge 0,\ \forall t \ge 0$.

Remark 3 For ETM (14), when the inequality in (14) holds, an event is triggered, and s_{k+1} is obtained. Since $\alpha > 0$ and $\gamma(t) \ge 0$, the inequality in (14) is evaluated and thus s_{k+1} is determined.

Remark 4 The method in this paper can be extended to estimate the state vectors of the following nonlinear time-delay system:

$$\dot{x}(t) = f(x(t),\,x(t-\tau_x),\,u(t),\,d(t)),\ \ t \ge 0, \tag{29}$$

$$x(\theta) = \phi(\theta),\ \ \theta \in [-\tau_x,\,0], \tag{30}$$

$$y(t) = Cx(t), \tag{31}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^p$ is the output vector, $d(t) \in \mathbb{R}^\ell$ is the external disturbance vector, $f(.)$ is a matrix function, $C \in \mathbb{R}^{p\times n}$ is a constant matrix, $\tau_x > 0$ is a known constant time delay, $\phi(\theta)$ is a continuous initial function.

For the first step, system (29)–(31) is predicted by the following RNN:

$$\begin{aligned}\dot{z}(t) = f_{rnn}(z(t),\, z(t-\tau_x),\, u(t)) = Az(t) + \Theta_z\sigma(z(t))\\ +\Theta_{\tau_x z}\sigma(z(t-\tau_x)) + \Theta_u u(t),\;\; t \geq 0,\end{aligned} \tag{32}$$

$$z(\theta) = \phi(\theta),\;\; \theta \in [-\tau_x,\, 0], \tag{33}$$

$$\overline{y}(t) = Cz(t), \tag{34}$$

where $z(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $\overline{y}(t) = \left[\overline{y}^1(t)\; \overline{y}^2(t)\; \overline{y}^3(t)\right]^T \in \mathbb{R}^{2n+m}$, $\overline{y}^1(t) = \begin{bmatrix}\overline{y}_1(t)\\ \vdots \\ \overline{y}_n(t)\end{bmatrix} = \begin{bmatrix}\sigma(z_1(t))\\ \vdots \\ \sigma(z_n(t))\end{bmatrix}$, $\overline{y}^2(t) = \begin{bmatrix}\overline{y}_{n+1}(t)\\ \vdots \\ \overline{y}_{2n}(t)\end{bmatrix} = \begin{bmatrix}\sigma(z_1(t-\tau_x))\\ \vdots \\ \sigma(z_n(t-\tau_x))\end{bmatrix}$, $\overline{y}^3(t) = \begin{bmatrix}\overline{y}_{2n+1}(t)\\ \vdots \\ \overline{y}_{2n+m}(t)\end{bmatrix} = \begin{bmatrix}u_1(t)\\ \vdots \\ u_m(t)\end{bmatrix}$, where $\sigma(\cdot)$ is the activation function satisfying the following inequality

$$|\sigma_i(v_1) - \sigma_i(v_2)| \leq \bar{\sigma}_i|v_1 - v_2|,\;\; \forall v_1,\, v_2 \in \mathbb{R}, \tag{35}$$

where $\bar{\sigma}_i > 0$ for $i = 1,\, 2,\, \ldots,\, n$ are positive scalars.

$A = -\operatorname{diag}\{a_1,\, a_2,\, \ldots,\, a_n\}$, $\begin{bmatrix}\Theta_z\\ \Theta_{\tau_x z}\\ \Theta_u\end{bmatrix} = \Theta$, $\Theta = \begin{bmatrix}\theta_1\\ \vdots \\ \theta_n\end{bmatrix} \in \mathbb{R}^{(2n+m)\times n}$ ($\theta_i = b_i\begin{bmatrix}w_{i1}\\ \vdots \\ w_{i(2n+m)}\end{bmatrix}^T$, $a_i > 0$, b_i are constants, for $i = 1,\, \ldots,\, n$, $j = 1,\, \ldots,\, n+m$, matrices w_{ij} are the weight connecting from the *jth* input to the *ith* neuron, which will be optimized during training.

We can determine optimal weighs a_i^* and θ_i^* for the RNN model (32) by solving the following ordinary least squares linear regression

$$(a_i^*,\, \theta_i^*) = \Lambda_i^* = \arg\min_{\Lambda_i} \frac{1}{2}\|S_i\Lambda_i - z_i\|^2, \tag{36}$$

where

$$S_i = \begin{bmatrix}[S_1]_i\;\; \overline{y}_1^\top\\ [S_2]_i\;\; \overline{y}_2^\top\\ \vdots \\ [S_N]_i\;\; \overline{y}_N^\top\end{bmatrix},\;\; z_i = \begin{bmatrix}f_i(s_1,\, s_{1\tau_x},\, u_1)\\ f_i(s_2,\, s_{2\tau_x},\, u_2)\\ \vdots \\ f_i(s_N,\, s_{N\tau_x},\, u_N)\end{bmatrix},\, \Lambda_i = \begin{bmatrix}a_i\\ \theta_i\end{bmatrix}.$$

For the second step, the following event-triggered state observer is proposed to estimate the state vector of the RNN model (32):

$$\begin{aligned}\dot{\hat{z}}(t) =& A\hat{z}(t) + \Theta_z\sigma(\hat{z}(t)) + \Theta_{\tau z}\sigma(\hat{z}(t-\tau_x))\\ &+ \Theta_u u(t) + K(\overline{y}(s_k) - C\hat{z}(s_k)),\;\; t \in [s_k,\, s_{k+1}),\end{aligned} \tag{37}$$

where $\hat{z}(t) \in \mathbb{R}^n$ is the estimate of $z(t)$, K is the gain matrix to be designed.

By following the proof of Theorem 1, we obtain the following theorem, which guarantees that the error dynamic system (37) is asymptotically stable:

Theorem 2 Given $\vartheta > 0$, system (37) is asymptotically stable if there exist $\mathcal{P} > 0$, $\mathcal{Q} > 0$, $\mathcal{R} > 0$, $\Gamma > 0$, Z, X, non-singular S, δ_1, $\delta_2 \in (0,\, \infty)$, such that for $\theta \in \{0,\, 1\}$:

$$\Delta^\star(\theta) = \begin{bmatrix}\Delta(\theta) & \nabla\\ * & -\operatorname{diag}(\delta_1 I_n,\, \delta_2 I_n)\end{bmatrix} < 0, \tag{38}$$

$$\Phi = \begin{bmatrix}\operatorname{diag}(\mathcal{R},\, \mathcal{R}) & Z\\ * & \operatorname{diag}(\mathcal{R},\, \mathcal{R})\end{bmatrix} > 0, \tag{39}$$

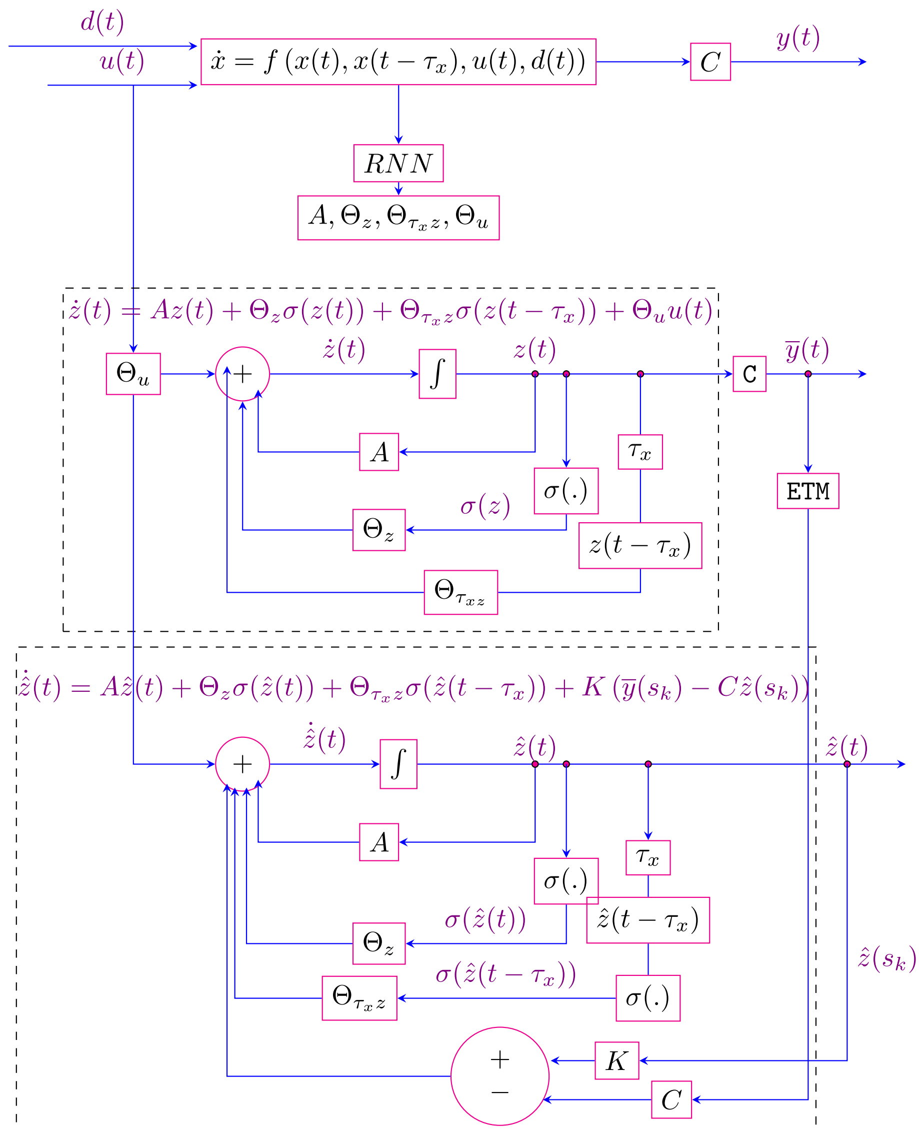

Fig. 2 Schematic of the event-triggered state observer (37)

where

$$\begin{aligned}
\Delta(\theta) &= \Delta_1(\theta) + \Delta_2(\theta) + \delta_1\bar{\sigma}^2_{\max}\epsilon_1^T\epsilon_1 + \delta_2\bar{\sigma}^2_{\max}\epsilon_8^T\epsilon_8,\ \bar{\sigma}_{\max} \\
&= \max\{\bar{\sigma}_1, \ldots, \bar{\sigma}_n\}, \\
\Delta_1(\theta) &= \ \mathrm{sym}\ \{\Psi_\theta^T P\Gamma\} + \epsilon_1^T\mathcal{Q}\epsilon_1 - \epsilon_3^T\mathcal{Q}\epsilon_3 + h^2\epsilon_4^T R_2\epsilon_4 - \Lambda^T\Phi\Lambda \\
&\quad + \mu\epsilon_2^T\Gamma\epsilon_2 + \mu\epsilon_7^T\Gamma\epsilon_7 - \ \mathrm{sym}\ \Big\{\big[\,\epsilon_1^T\ \epsilon_4^T\,\big] \\
&\quad\quad \begin{bmatrix}\vartheta S\\ S\end{bmatrix}\epsilon_4\Big\}, \\
\Delta_2(\theta) &= \ \mathrm{sym}\ \Big\{\big[\,\epsilon_1^T\ \epsilon_4^T\,\big] \\
&\quad\quad \begin{bmatrix}\vartheta S\\ S\end{bmatrix}A\epsilon_1 - \big[\,\epsilon_1^T\ \epsilon_4^T\,\big]\begin{bmatrix}\vartheta X\\ X\end{bmatrix}C\epsilon_2 - \big[\,\epsilon_1^T\ \epsilon_4^T\,\big]\begin{bmatrix}\vartheta X\\ X\end{bmatrix}C\epsilon_7\Big\}, \\
\Psi_\theta &= \big[\,\epsilon_1^T\ \theta h\epsilon_5^T + (1-\theta)h\epsilon_6^T\,\big]^T,\ \Gamma = \big[\,\epsilon_4^T\ (\epsilon_1^T - \epsilon_3^T)\,\big]^T, \\
\Lambda &= \big[\,\Lambda_1\ \Lambda_2\ \Lambda_3\ \Lambda_4\,\big]^T,\ \Lambda_1 = \big[\,(\epsilon_1 - \epsilon_2)^T\,\big], \\
\Lambda_2 &= \big[\,\sqrt{3}(\epsilon_1 + \epsilon_2 - 2\epsilon_5)^T\,\big],\ \Lambda_3 = \big[\,(\epsilon_2 - \epsilon_3)^T\,\big],
\end{aligned}$$

$$\Lambda_4 = \left[\, \sqrt{3}(\epsilon_2 + \epsilon_3 - 2\epsilon_6)^T \,\right],$$
$$\nabla = \left[\, \epsilon_1^T \ \epsilon_4^T \,\right] \begin{bmatrix} \vartheta S \\ S \end{bmatrix} (\Theta_z h_1 + \Theta_{\tau z} h_2),$$
$$h_1 = \left[\, I_n \ 0_{n\times n} \,\right],\ h_2 = \left[\, 0_{n\times n} \ I_n \,\right],$$
$$\epsilon_i = \left[\, 0_{n\times(i-1)n} \ I_n \ 0_{n\times(7-i)n} \ 0_{n\times n} \,\right] \in \mathbb{R}^{n\times(7n+n)},\ i = 1, \ldots, 7,$$
$$\epsilon_8 = \left[\, 0_{n\times 7n} \ I_n \,\right] \in \mathbb{R}^{n\times(7n+n)}.$$

The observer gain matrix K is obtained as

$$K = S^{-1}X. \tag{40}$$

4 Event-triggered state estimation for the PMSM

We now apply the obtained results in Sect. 3 to estimate the electrical angular velocity, the electrical angle, and the currents of the following permanent magnet synchronous motor [7]:

$$\begin{aligned}\dot{\omega}_r(t) = &\frac{3n_p^2}{2J}\Big(\psi_r + (L_d - L_q)i_d(t)\Big)i_q(t) \\ &- \frac{n_p}{J}s_L - \frac{1}{J}\mathsf{B}\omega_r(t),\end{aligned} \tag{41}$$

$$\dot{\theta}_r(t) = \omega_r(t), \tag{42}$$

$$\begin{aligned}\dot{i}_q(t) = &-\frac{R_s}{L_q}i_q(t) - \omega_r(t)\frac{L_d}{L_q}i_d(t) \\ &- \omega_r(t)\frac{\psi_r}{L_q} + \frac{1}{L_q}u_q(t),\end{aligned} \tag{43}$$

$$\dot{i}_d(t) = -\frac{R_s}{L_d}i_d(t) + \omega_r(t)\frac{L_q}{L_d}i_q(t) + \frac{1}{L_d}u_d(t), \tag{44}$$

where R_s is the stator resistance (Ω), $u_d(t)$, $u_q(t)$, $i_d(t)$, $i_q(t)$, L_d and L_q are the $d-q$ axis stator voltages (V), currents (A) and inductances (Wb), respectively, ψ_r is the amplitude of the permanent magnet flux linkage (Wb), $\omega_r(t)$ and $\theta_r(t)$ are the electrical angular velocity (rad/s) and the electrical angle (rad), n_p is the number of pole pairs, s_L is the load torque (N.m), J and B_b are the total moment of inertia (kg.m^2) and the viscous friction coefficient (Nm.s/rad).

Clearly, the PMSM (41)–(44) is in the form of system (1)–(3). It follows from [10] that assumptions (A_1), (A_2), and (A_3) hold and the error between the state of (1)–(3) and the recurrent neural networks (4)–(6) satisfies the following inequality:

$$||e(t)|| \leq \frac{\frac{n_p}{J}\bar{d} + \bar{\omega}}{\gamma}(e^{\gamma t} - 1),\ \ t > 0, \tag{45}$$

where $e(t) = x(t) - z(t)$, $\bar{\omega}$ is a positive number, and

$$\gamma = \sqrt{\max\{4M\frac{L_d^2}{L_q^2},\, 2M(\frac{9n_p^4(L_d - L_q)^2}{4J^2} + \frac{L_d^2}{L_q^2})\}}$$

$M \in (0,\, \infty)$ such that the states of the permanent magnet synchronous motor operate in a bounded region $\mathcal{R} = \{z \in \mathbb{R}^4 | |z_i| \leq M\}$. By using the RNN (4)–(6) and the event-triggered state observer (13), the trajectory of the PMSM is estimated.

5 Conclusion

We have solved the problem of estimating the states of nonlinear systems. A RNN model which predicts the nonlinear systems and a dynamic event-triggered state observer for this model have been derived. The obtained theoretical analysis is applied to estimate the electrical angular velocity, the electrical angle, and the currents of the permanent magnet synchronous motor. Numerical results have been provided to demonstrate the merit of the proposed method. Further work is required to consider the event-triggered state estimation problem for time-delay nonlinear systems with external disturbances in the outputs. Also, extending the method in this paper to solve the problem of estimating the state vectors of nonlinear fractional-order systems and nonlinear interconnected systems are interesting problems for future research.

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Data availability No data associated in the manuscript.

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