ON STRONGLY CONVEX FUNCTIONS AND SET-VALUED MAPS

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Let D be a convex subset of a normed space and $c > 0$. A function $f : D \to \mathbb{R}$ is called *strongly convex* with modulus c if

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) - ct(1-t)\|x-y\|^2$$

for all $x, y \in D$ and $t \in [0,1]$; f is called *strongly midconvex* with modulus c if

$$f\Big(\frac{x+y}{2}\Big) \leq \frac{f(x)+f(y)}{2} - \frac{c}{4}\|x-y\|^2, \quad x, y \in D.$$

Strongly convex functions are useful in optimization theory and mathematical economics. Many properties and applications of them can be found in the literature. In my talk some results on strongly convex functions and related classes of functions obtained by the author with co-authors in the last few years are presented. In particular, discrete and integral Jensen-type inequalities and a Hermite–Hadamard–type inequality for strongly convex functions are obtained. Counterparts of the classical Bernstain–Doetsch and Sierpiński theorems for strongly midconvex functions are given. New characterizations of inner product spaces involving strong convexity are obtained. A representation of strongly Wright-convex functions and a characterization of functions generating strongly Schur-convex sums are presented. Finally, some properties of strongly convex and strongly midconvex set-valued maps are presented.

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