

# TÓM TẮT CÁC BÁO CÁO

# On the $(1 - C_2)$ condition

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Throughout this talk every ring is associative with identity, and all modules are unital right modules. In [1], Muller and Rizvi considered the following condition for an  $R$ -module  $M$ :

$(1 - C_2)$ : *Every uniform submodule isomorphic to a direct summand of  $M$  is itself a direct summand of  $M$ .*

A module  $M$  is called a  $(1 - C_2)$ - module if it satisfies the condition  $(1 - C_2)$ . The module  $M$  is a 1-continuous module if it satisfies  $(1 - C_2)$  and uniform extending. And, a module  $M$  is called a strongly 1-continuous module if it satisfies  $(1 - C_2)$  and CS. In this talk, we use these classes of modules to describe some classes of rings, e.g., as semiprime rings and  $SI$ -,  $V$ -,  $QF$ -rings.

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# The influence of idempotents on the structure of rings

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The Pierce decomposition associated to an idempotent  $e^2 = e \in R$  can be considered as a generalized matrix ring structure

$$\begin{pmatrix} eRe & eR(1-e) \\ (1-e)Re & (1-e)R(1-e) \end{pmatrix}$$

on  $R$ . Therefore each idempotent can be associated to a Morita context

$$\{A, B, {}_A M_B, {}_B N_A, [-, -] : M \otimes_B N \rightarrow A, (-, -) : N \otimes_A M \rightarrow B\}.$$

This shows that by specializing bilinear products  $[-, -]$  and  $(-, -)$  one can obtain more information about the structure of  $R$ . This is a topic of a joint work together with G. Birkenmeier and L. van Wyk.

# Index of reducibility of Noetherian modules

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Let  $M$  be a finitely generated module over a Noetherian ring  $R$  and  $N$  a submodule. The index of reducibility  $\text{ir}_M(N)$  is the number of irreducible submodules that appear in an irredundant irreducible decomposition of  $N$  (this number is well defined by a classical result of Emmy Noether). Then the main results of this talk are: (1)  $\text{ir}_M(N) = \sum_{\mathfrak{p} \in \text{Ass}_R(M/N)} \dim_k \text{Soc}(M/N)_{\mathfrak{p}}$ ; (2) For an irredundant primary decomposition of  $N = Q_1 \cap \dots \cap Q_n$ , where  $Q_i$  is  $\mathfrak{p}_i$ -primary, then  $\text{ir}_M(N) = \text{ir}_M(Q_1) + \dots + \text{ir}_M(Q_n)$  if and only if  $Q_i$  is a  $\mathfrak{p}_i$ -maximal embedded component of  $N$  for all embedded associated prime ideals  $\mathfrak{p}_i$  of  $N$ ; (3) For an ideal  $I$  of  $R$  there exists a polynomial  $\text{Ir}_{M,I}(n)$  such that  $\text{Ir}_{M,I}(n) = \text{ir}_M(I^n M)$  for  $n \gg 0$ . Moreover,  $\text{bight}_M(I) - 1 \leq \deg(\text{Ir}_{M,I}(n)) \leq \ell_M(I) - 1$ ; (4) If  $(R; \mathfrak{m})$  is local,  $M$  is Cohen-Macaulay if and only if there exist an integer  $l$  and a parameter ideal  $\mathfrak{q}$  of  $M$  contained in  $\mathfrak{m}^l$  such that  $\text{ir}_M(\mathfrak{q}M) = \dim_k \text{Soc}(H_{\mathfrak{m}}^d(M))$ , where  $d = \dim M$ .

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<sup>†</sup> Joint work with Pham Hung Quy and Hoang Le Truong.

# A note on hereditary Noetherian serial rings

Phan Dan

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A ring  $R$  is called right (left) hereditary if each right (left) ideal of  $R$  is projective. A right and left hereditary ring is called hereditary. A module is called uniserial if the set of all its submodules is linearly ordered. A module is said to be serial if it is a direct sum of uniserial modules. A ring  $R$  is called right (left) serial if  $R$  is serial as a right (left)  $R$ -module. A right and left serial ring is called serial. It is well-known that every hereditary serial ring is (two sided) noetherian, and such a ring is a direct sum of artinian rings and prime rings [3].

It was shown by S. Singh [1] that a semiprime serial right (or left) noetherian ring is noetherian.

A. A. Tuganbaev has shown that a semiprime ring  $R$  is hereditary serial iff  $R$  is right serial left noetherian iff  $R$  is left serial right noetherian [2].

The purpose of this note is to investigate a class of right nonsingular right Goldie rings and to generalize the above results.

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# On weakly locally finite division rings

**Trinh Thanh Deo**

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In this talk, we give the definition of weakly locally finite division rings and show that the class of these rings strictly contains the class of locally finite division rings. Further, we study multiplicative subgroups in these rings. Some skew linear groups are also considered.

# Free subgroups in division rings

**Bui Xuan Hai**

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Let  $D$  be a division ring with center  $F$ , and  $D^*$  be the multiplicative group of  $D$ . The existence of non-cyclic free subgroups in  $D^*$  has been posed as a conjecture by Lichtman when he studied the analogue of Tits' Alternative for matrix rings over division rings. Further, several authors consider the existence of other free objects in division rings. In this talk, we discuss this problem. In particular, the existence of free subgroups in maximal subgroups of  $D^*$  is also considered.

# An addition to Artin-Wedderburn's Theorem and a weaker form of $p$ -injective modules

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In this talk, we introduce a weaker form of  $p$ -injective modules,  $M$ -cpp-injective module. Semisimple rings are characterized by cpp-injective modules. By this way, we give an additional characterization of semisimple rings to the **Artin-Wedderburn's well-known theorem**. Moreover, we obtain good results similarly to principally injective rings by Nicholson and Yousif. Finally, we describe several indications regarding annihilator conditions among  $M$ -cpp-injective modules likely Ikeda-Nakayama modules.

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<sup>†</sup> Joint work with Le Ngoc Hoa, Do Van Thuat and Nguyen Van Sanh.



# On Weakly $D2$ modules

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Modules have the  $D2$ -condition over general rings are considered and generalizations of  $D2$ -condition are defined. In order to study these module classes, we generalize the notions of the  $D2$ -condition. Here a module  $M_R$  is called *weakly  $D2$*  for every  $s \in S$  and  $s \neq 0$ , there exists  $n \in \mathbb{N}$  such that  $s^n \neq 0$  and if  $\text{Im}(s^n)$  is a direct summand of  $M$ , implies that  $\text{Ker}(s^n)$  is a direct summand of  $M$ . Some characterizations are given for right weakly  $D2$  rings and their connections with von Neumann regular and right  $d$ -Rickart rings, which asserts that  $R$  is von Neumann regular if and only if  $R$  is right  $d$ -Rickart right weakly  $D2$ .

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† Joint work with Truong Cong Quynh, M. Tamer Koşan and Serap Şahinkaya.

# On generalization IFP modules and rings

Truong Thi Hien

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A submodule  $X$  of a right  $R$ -module  $M$  is said to have "generalization of insertion of factor property" (briefly, G-IFP submodule) if for any endomorphism  $\varphi \in S = \text{End}(M)$  and  $m \notin X$ , if  $\varphi(m) \in X$ , then there is  $m' \notin X$  such that  $\varphi S(m') \subset X$ . A right  $R$ -module  $M$  is called G-IFP module if  $0$  is a g-IFP submodule of  $M$ . The concept of G-IFP modules is a generalization of G-IFP rings as well as IFP modules. In this thesis, we investigate basic properties and relationships between G-IFP modules and their endomorphism rings. Several known results related to G-IFP rings and IFP modules can be obtained as corollaries of our results.

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<sup>†</sup> Joint work with Nguyen Van Sanh and Nguyen Dang Hoa Nghiem .

# Morita equivalence and simple semirings

Tran Giang Nam

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In this talk, a semiring is an algebraic structure similar to a ring, but its additive reduct is a commutative idempotent monoid. Finite congruence-simple semirings were recently applied for constructing new public-key cryptosystems, which originated from the work of Monico, Maze and Rosenthal [3, 4, 6]. There have been several studies on the structure of congruence-simple semirings, e.g., in [1, 2, 5, 7]. These studies bring naturally to the following question: Do there exist finite ideal-simple semirings that are not congruence-simple? The purpose of this talk is to consider this question. This work was done in collaboration with Prof. Sergey N. Il'in.

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# Modules are invariant under endomorphisms of their envelopes

Truong Cong Quynh  
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Let  $M$  be a right  $R$ -module.  $M$  is called CS (or extending) if every submodule of  $M$  is essential in a direct summand of  $M$ . It is well-known  $M$  is extending if and only if for any idempotent  $g$  in  $\text{End}(E(M))$ , there exists an idempotent  $f$  in  $\text{End}(M)$  such that the intersection of  $g(E(M))$  with  $M$  is  $f(M)$ .  $M$  is called quasi-continuous if  $M$  is invariant under idempotents of  $\text{End}(E(M))$ . The concept of extending module and quasi-continuous module is studied by many authors. Some their characterizations are obtained. Moreover, some their applications used studying structure of some classes of rings. In other words, the class of extending modules and quasi-continuous modules play an important role in the theory of ring. In this talking, I want to talk about the concept of extending module and quasi-continuous module by other way definition. We used the concept of envelope to define this definitions. The our purpose study the new concept of extending module and quasi-continuous module in generality and then the old result of extending module and quasi-continuous module are corollaries.

# A characterization of co-Harada ring

Le Duc Thoang  
Phu Yen University

A ring  $R$  is called right co-Harada if every non-cosmall right  $R$ -module contains a non-zero projective direct summand and  $R$  satisfies the ACC on right annihilators. In this paper we give a characterization of co-Harada rings via left perfect rings.

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# On $QF$ rings and related modules

**Le Van Thuyet**

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A ring  $R$  is called  $QF$  if  $R$  is right (or left) self-injective and satisfies ACC on right (or left) annihilators. In this talk, we obtain some properties of a small injective module  $M$ , i.e., every homomorphism from a small right ideal of  $R$  can be extended to a homomorphism from  $R$  to  $M$ . A ring  $R$  is called right small injective if the right  $R$ -module  $R_R$  is small injective. And then we give some characterizations of  $QF$ , right  $PF$  rings via the classes of above modules.

# Some Properties of $e$ -Supplemented and $e$ -Lifting Modules

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A module  $M$  is called  $\delta$ -supplemented if every submodule  $N$  of  $M$ , there exists  $L \leq M$  such that  $M = N + L$  and  $N \cap L \leq_{\delta} L$ . A module  $M$  is called  $\delta$ -lifting if for any  $N \leq M$ , there exists a decomposition  $M = A \oplus B$  such that  $A \leq N$  and  $N \cap B \leq_{\delta} M$ . In this talk, we study  $e$ -supplemented modules and  $e$ -lifting modules which is generalized  $\delta$ -supplemented modules and  $\delta$ -lifting modules. Some properties of these modules are considered. Moreover, some new characterizations of Artinian (resp., Noetherian)  $\text{Rad}_e(M)$  module are studied via chain condition on  $e$ -small submodules.

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# Diamond Lemma and Groebner basis for Leavitt path algebras

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We consider the Groebner basis theory for free algebra over a field and prove the Diamond lemma of Bergman. We then apply in the Leavitt path algebra to construct their Groebner bases. Also, some applications for Noetherian Leavitt path algebras are considered.



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